Introduction to Nuclear and Particle Physics

Lesson 6

elastic scattering

Warm-up question 1

We want to study the electric charge distribution of the Ca48 nucleus. For this purpose we can measure:

A) Cross sections of Rutherford scattering of α particles on Ca48.

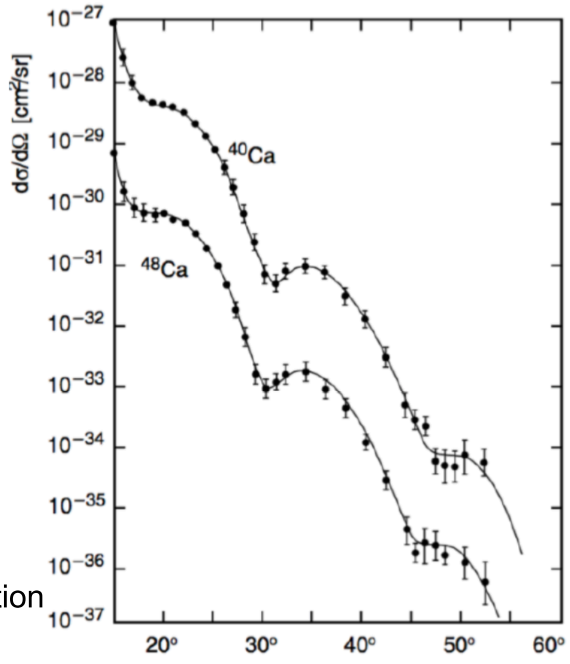
B) Cross sections of elastic electron scattering on Ca48.

C) Cross sections of inelastic electron scattering on Ca48.

D) None of them.

Warm-up question 2

Have a look at the differential cross sections in the right. Which statements are correct?



A) The two cross sections are obtained by performing
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B) The two cross sections are obtained from electron scattering
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C) The oscillating shape indicates that the nuclear charge has a "Fermi distribution" (sphere with a homogeneously distributed charge and diffuse radius)

D) The oscillation shape indicates that the nuclear charge has an exponential distribution

E) The oscillating shape indicates that the nuclear charge distribution is point-like.
The various peaks indicate various resonances excited by the inelastic collision.

Plan for today

Mott scattering

Rutherford scattering

Elastic collisions

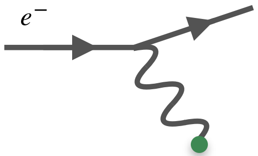
Form factors

Energy scale

Electron scattering and structure determination

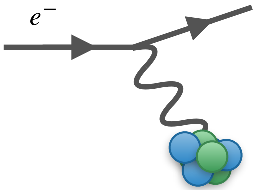
Elastic domain

$\lambda \approx 1\ \text{fm}$



point charge

Mott



charge distribution

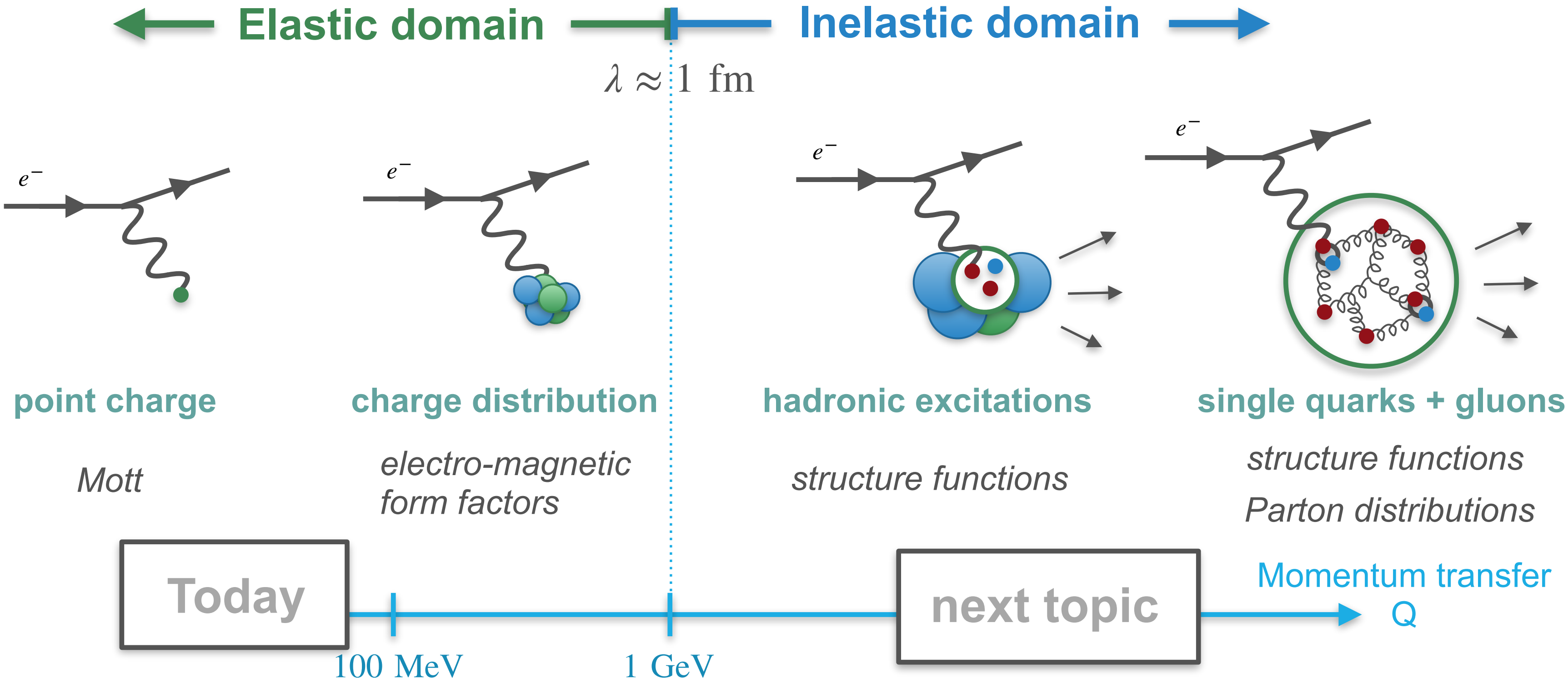
electro-magnetic form factors

Momentum transfer Q

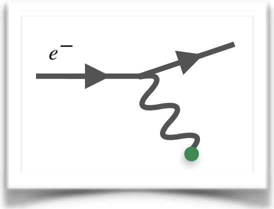
100 MeV

1 GeV

Electron scattering and structure determination



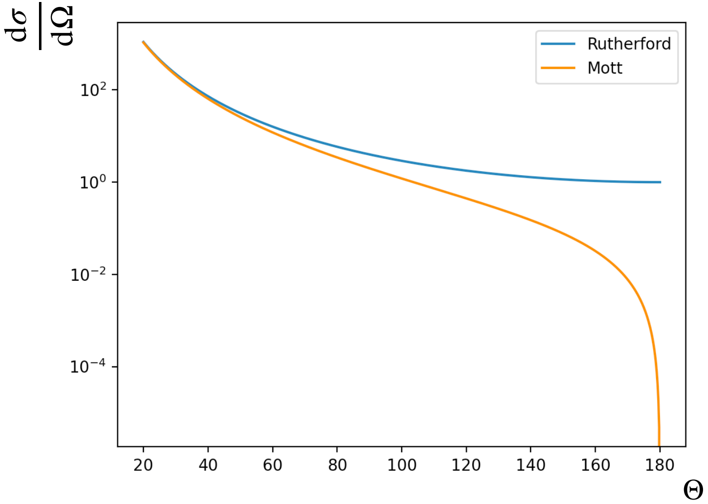
Low-energy elastic scattering of e-



Rutherford

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_R \sim \frac{\alpha^2}{E^2 \sin^4 \frac{\theta}{2}}$$

without spin



Mott

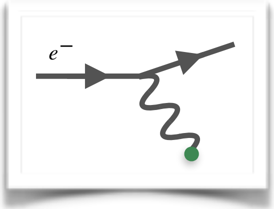
$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_M = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_R \left(1 - \beta^2 \sin^2 \frac{\theta}{2}\right)$$

correction for spin for $\beta \to 1$

Assumptions:

- point-like projectiles
- only electro-magnetic interaction

Low-energy elastic scattering of e-



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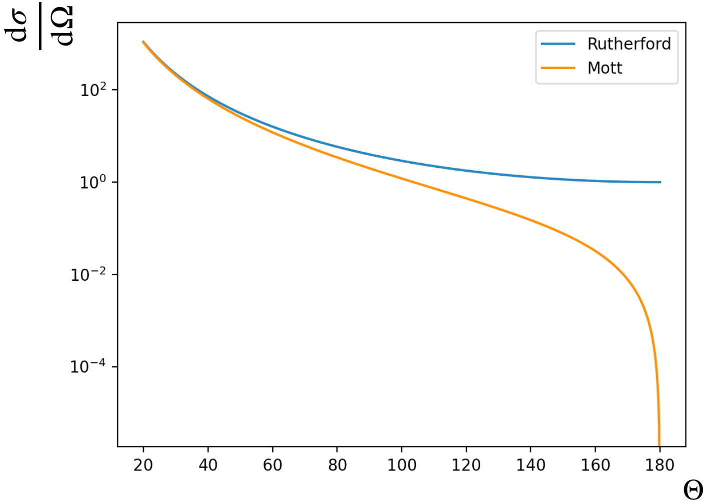

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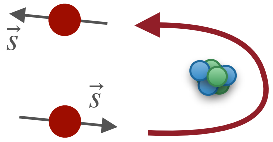
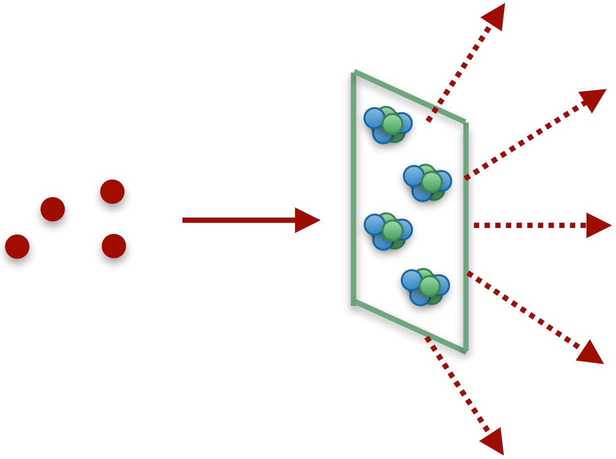


“helicity conservation”: spin || direction of flight

Assumptions:
- point-like projectiles
- only electro-magnetic interaction

Rutherford simulations



Main result of historic Rutherford experiment:

Nearly all mass of atoms is focused in tiny nuclei.
We want to know more!

https://phet.colorado.edu/sims/html/rutherford-scattering/latest/rutherford-scattering_en.html

Additional remark on electron screening

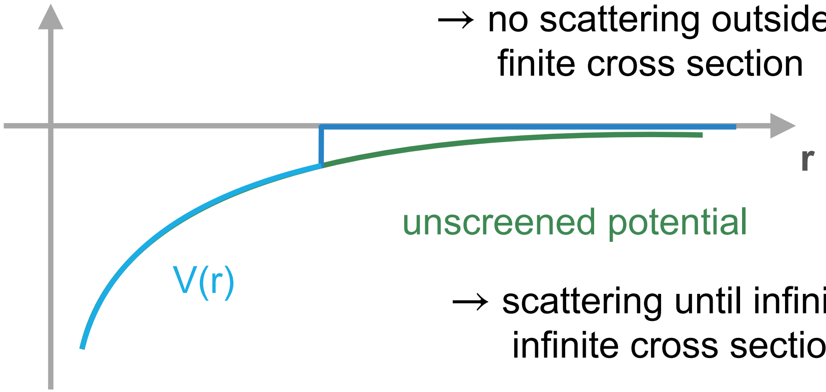
The Rutherford cross section $\theta = 0$. What physical explanation is behind that?

Answer:

The formula does not take into account the screening of the nuclear charge by electrons.

screened potential

→ no scattering outside of atom,
finite cross section



unscreened potential

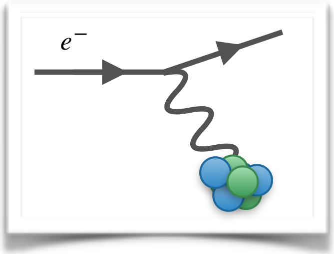
→ scattering until infinity,
infinite cross section

$$\frac{d\sigma}{d\Omega} \sim \langle f | H_{int} | i \rangle^2$$

includes V

ENGAGING PHYSICS TUTORING EPT

Nuclear form factors



How to correct for the shape of the nucleus?

Most obvious approach:

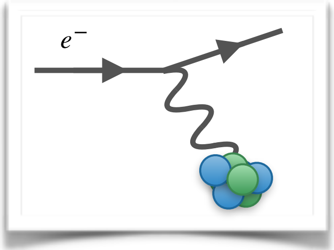
$$\left(\frac{d\sigma}{d\Omega}\right)_{Exp} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left|F\left(q^2\right)\right|^2$$

(spherically symmetric)

Remember Fermi's Golden Rule!

$$\left(\frac{d\sigma}{d\Omega}\right)_{Exp} \sim \left|M_{fi}\right|^2$$

Nuclear form factors



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$$\left(\frac{d\sigma}{d\Omega}\right)_{Exp} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left|F\left(q^2\right)\right|^2 \longleftrightarrow \left(\frac{d\sigma}{d\Omega}\right)_{Exp} \sim \left|M_{fi}\right|^2$$

(spherically symmetric)

connected with charge distribution

$$f(r) = \frac{1}{(2\pi)^3} \int e^{-\frac{i\vec{q}\cdot\vec{r}}{\hbar}} F\left(q^2\right)$$

Analogy: double slit interference

slit geometry ⟷ **interference pattern**

Form factors: Why electrons?

In his original scattering experiment, Ernest Rutherford studied the scattering of α particles in gold.

Why do most modern scattering experiments measure electro-magnetic form factors with electrons instead?

A) At the corresponding energies electrons are easier to produce.

B) Using α particles, inelastic nuclear interactions occur and disturb the measurement of the electro-magnetic structure.

C) It is not possible to resolve the nuclear structure using α particles even with kinetic energies of several 100 MeV because their deBroglie wave length is too long.

D) Electrons are point-like and therefore an ideal probe.

Electron scattering experiment in Mainz **[MAMI]**

Form factors: Why electrons?

In his original scattering experiment, Ernest Rutherford studied the scattering of α particles in gold.

Why do most modern scattering experiments measure electro-magnetic form factors with electrons instead?

α can also be accelerated!

A) ~~At the corresponding energies electrons are easier to produce.~~

B) Using α particles, inelastic nuclear interactions occur and disturb the measurement of the electro-magnetic structure. ✓

C) ~~It is not possible to resolve the nuclear structure using α particles even with kinetic energies of several 100 MeV because their deBroglie wave length is too long.~~

D) Electrons are point-like and therefore an ideal probe. ✓

Electron scattering experiment in Mainz **[MAMI]**

$\lambda = \frac{h}{p}$ with $|p| = \sqrt{E_{kin}^2 + 2m_\alpha E_{kin}}$

$\Rightarrow$ For similar E_{kin} the α has even shorter wavelength than electrons!

Question scattering cross section

Compare the measured data in the plot on the right with the prediction for Screened Mott.

What can we learn from the discrepancy?

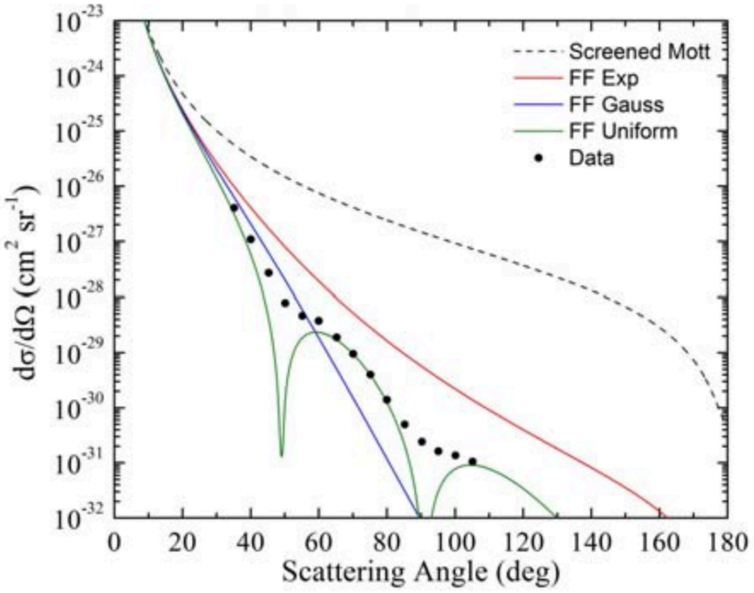
A) The Mott equation does not account for the helicity conservation sufficiently.

B) The nuclear charge is not concentrated in one single point.

C) The discrepancy can be explained by inelastic processes happening additionally.

D) The detection efficiency was really bad at large angles.

Scattering of e$^-$ (183 MeV) in Indium



[A Bagulya *et al* 2017 *J. Phys.: Conf. Ser.* **898** 042032]

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it does!

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Inelastic Processes not important at this energy

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Cross section does not include detection efficiency.

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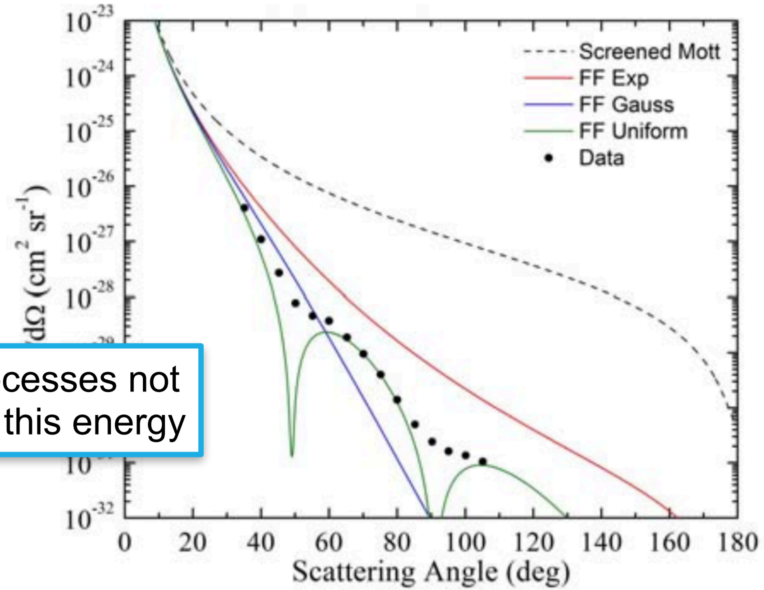

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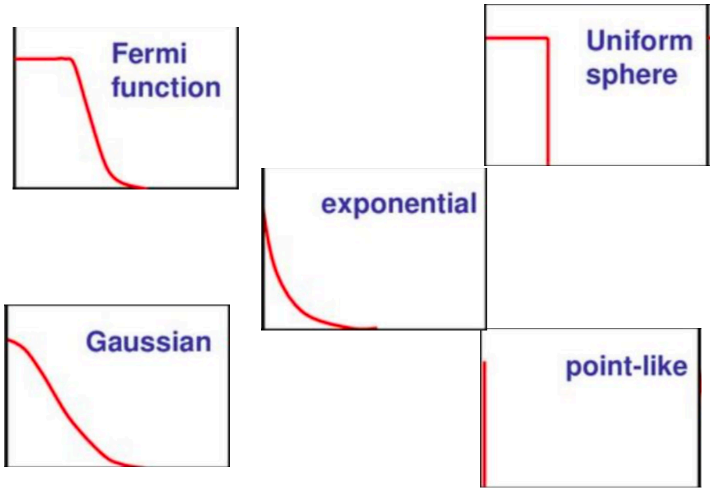
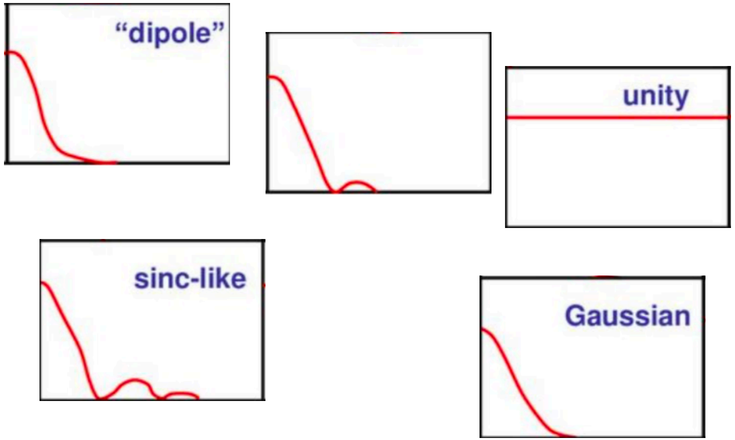
Group activity

Match the form factors to the charge distributions!

Charge distributions



Form factors



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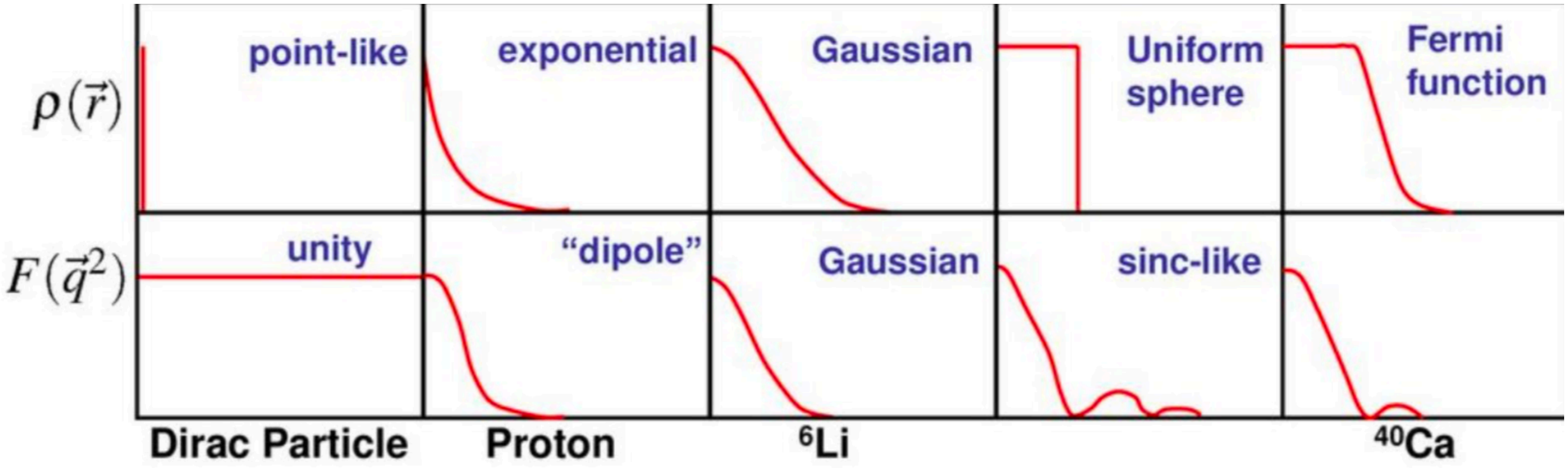
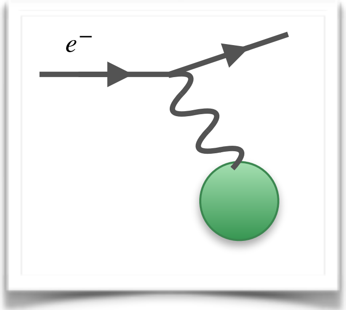


Rosenbluth: Structure of single nucleons

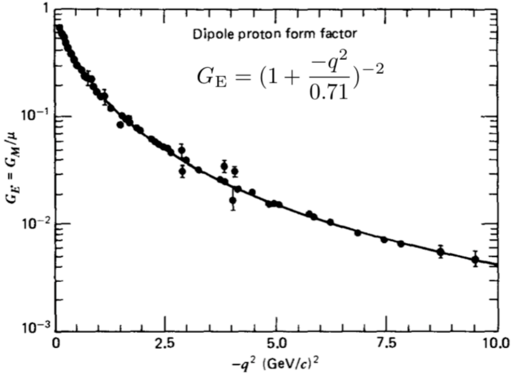


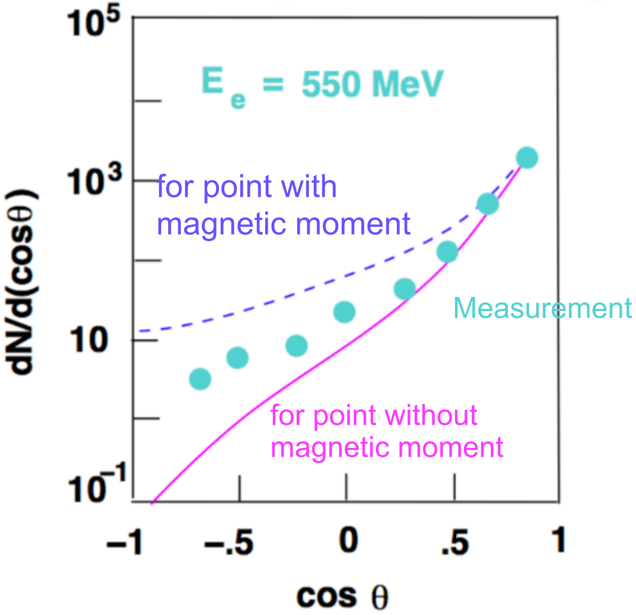
At high momentum transfer, we see both electric and magnetic properties of the nucleon.

$$\left(\frac{d\sigma}{d\Omega}\right)_{Exp} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left[\frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2\frac{\theta}{2}\right]$$

Example:

Proton electric form factor





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disturbing nuclear effects

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Inelastic collision: something changes with electron and/or nucleus

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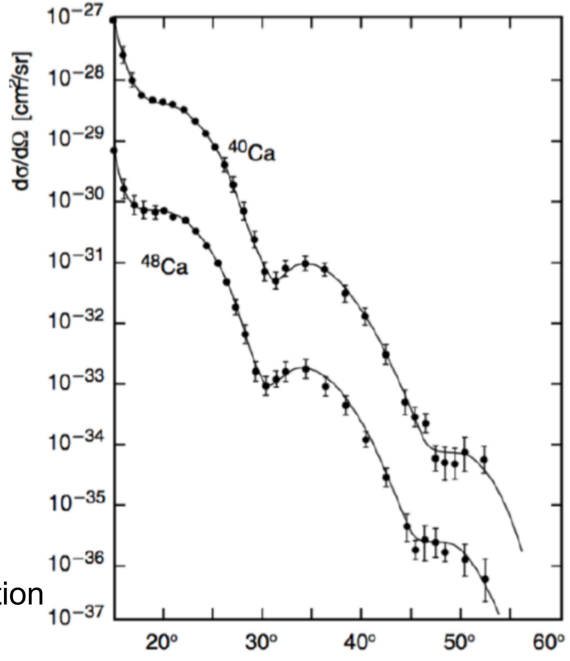

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