3.4 MATERIAL PROPERTIES AND THEIR UNITS

Each material can be thought of as having a set of attributes or properties. The combination that characterizes a given material is its *property profile*. Property profiles are assembled by systematic testing. In this section we scan the nature of the tests and the definition and units of the properties (see Table 3.1). Property values are listed in Appendix A. Units are given here in the SI system. Conversion factors to other systems are printed on the inside front and back cover of the book.

General properties

The *density*, ρ (units: kg/m^3), is the mass per unit volume. We measure it today as Archimedes did: by weighing in air and in a fluid of known density.

The *price*, C_m (units: \$/kg), spans a wide range. Some cost as little as \$0.2/kg, others as much as \$1,000/kg. Prices, of course, fluctuate, and they depend on the quantity you want and on your status as a "preferred customer" with your chosen vendor. Despite this uncertainty, it is useful to have an approximate price in the early stages of material selection.

Mechanical properties

The *elastic modulus*, E (units: GPa or GN/m^2), is the slope of the initial, linear-elastic, part of the stress-strain curve (Figure 3.3). Young's modulus, E, describes response to tensile or compressive loading; the shear modulus, G, describes response to shear loading; and the bulk modulus, K, describes the response to hydrostatic pressure. Poisson's ratio, ν, is the negative of the ratio of the lateral strain, ε_2, to the axial strain, ε_1, in axial loading:

$$\nu = -\frac{\varepsilon_2}{\varepsilon_1}$$
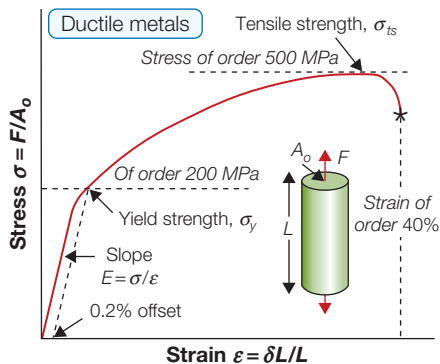
FIGURE 3.3

The stress-strain curve for a metal, showing the modulus, E, the 0.2% yield strength, σ_y, and the ultimate strength, σ_{ts}.

In reality, moduli measured as slopes of stress-strain curves are inaccurate, often low by a factor of 2 or more, because of contributions to the strain from inelasticity, creep, and other factors. Accurate moduli are measured dynamically: by exciting the natural vibrations of a beam or a wire or by measuring the velocity of sound waves in the material.

In an isotropic material, the moduli are related in the following ways:

$$E = \frac{3G}{1 + G/3K} \quad G = \frac{E}{2(1+\nu)} \quad K = \frac{E}{3(1-2\nu)} \tag{3.1}$$

Commonly,

$$\nu \approx 1/3$$

when

$$G \approx \frac{3}{8}E$$

and

$$K \approx E \tag{3.2a}$$

Elastomers are exceptional. For these,

$$\nu \approx 1/2$$

when

$$G \approx \frac{1}{3}E$$

and

$$K \gg E \tag{3.2b}$$

Data sources like those described in Appendix D list values for all four moduli. In this book we examine data for E; approximate values for the others can be derived from the (3.2) equations when needed.

Estimating moduli

Young's modulus E for copper is 124 GPa; its Poisson's ratio ν is 0.345. What is its shear modulus, G?

Answer
Inserting the values for E and ν in the central equation (3.1) gives G = 46.1 GPa. The measured value is 45.6 GPa, a difference of only 1%.

The *strength*, σ_f (units: MPa or MN/m^2), of a solid requires careful definition. For metals, we identify σ_f with the 0.2% offset yield strength σ_y (see Figure 3.3), that is, the stress at which the stress-strain curve for axial loading deviates by a strain of 0.2% from the linear-elastic line. It is the same in tension and compression. For polymers, σ_f is identified as the stress at which the stress-strain curve becomes markedly nonlinear, at a strain typically of 1% (Figure 3.4). This may be caused by shear yielding: the irreversible slipping of molecular chains; or it may be caused by crazing: the formation of low-density, crack-like volumes that scatter light, making the polymer look white. Polymers are a little stronger ($\approx$ 20%) in compression than in tension.

Strength, for ceramics and glasses, depends strongly on the mode of loading (Figure 3.5). In tension, "strength" means the fracture strength, σ_t. In compression it means the crushing strength σ_c, which is much greater; typically

$$\sigma_c = 10 \text{ to } 15\,\sigma_t \tag{3.3}$$

When a material is difficult to grip, as is a ceramic, its strength can be measured in bending. The *flexural strength* or *modulus of rupture*, σ_{flex} (units: MPa) is the maximum surface stress in a bent beam at the instant of failure (Figure 3.6). One might expect this to be the same as the strength measured in tension, but for ceramics it is greater by a factor of about 1.3 because the volume subjected to this maximum stress is small and the probability of a large flaw lying in it is small also; in simple tension all flaws see the same stress.
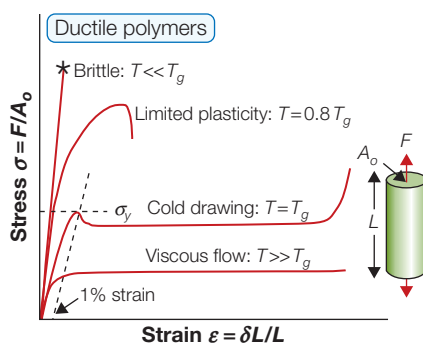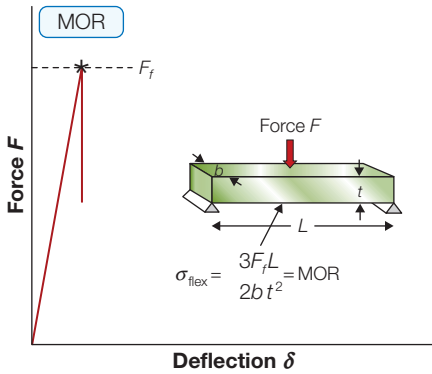

FIGURE 3.4
Stress-strain curves for a polymer below, at, and above its glass transition temperature, T_g.
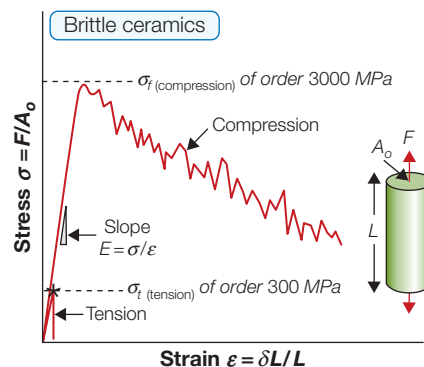
FIGURE 3.5
Stress-strain curves for a ceramic in tension and in compression. The compressive strength, σ_c, is 10 to 15 times greater than the tensile strength, σ_t.

FIGURE 3.6

The modulus of rupture (MOR) is the surface stress at failure in bending. It is equal to, or slightly larger than, the failure stress in tension.

The strength of a composite is best defined by a set deviation from linear-elastic behavior; often an off-set of 0.5% is taken. Composites that contain fibers, including natural composites such as wood, are a little weaker (up to 30%) in compression than tension because fibers buckle. In subsequent chapters, σ_f for composites means the tensile strength.

Strength, then, depends on material class and on mode of loading. Other modes of loading are possible: shear, for instance. Yield under multiaxial loads is related to that in simple tension by a *yield function*. For metals, the Von Mises yield function is a good description:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_f^2 \tag{3.4}$$

where σ_1, σ_2, and σ_3 are the principal stresses, positive when tensile; σ_1, by convention, is the greatest or most positive; σ_3, the smallest or least positive. For polymers the yield function is modified to include the effect of pressure:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_f^2 \left(\frac{1+\beta p}{K}\right)^2 \tag{3.5}$$

where K is the bulk modulus of the polymer, $\beta \approx 2$ is a numerical coefficient that characterizes the pressure dependence of the flow strength, and the pressure p is defined by

$$p = -\frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$$

For ceramics, a Coulomb flow law is used:

$$\sigma_1 - B\sigma_2 = C \tag{3.6}$$

where B and C are constants.

The *tensile* (or *ultimate*) *strength* σ_{ts} (units: MPa) is the nominal stress at which a round bar of the material, loaded in tension, separates (Figure 3.3). For brittle solids—ceramics, glasses, and brittle polymers—it is the same as the failure strength in tension. For metals, ductile polymers, and most composites, it is greater than the yield strength, σ_y, by a factor of between

Using yield functions

A metal pipe of radius r and wall thickness t carry an internal pressure p. The pressure generates a circumferential wall stress of $\sigma_1 = pr/t$, an axial wall stress $\sigma_2 = pr/2t$. At what pressure will the pipe first yield?

Answer

Setting $\sigma_2 = \sigma_1/2$, $\sigma_3 = 0$ and $\sigma_f = \sigma_y$ in Equation (3.4) gives the yield condition $\sigma_1 = (2/\sqrt{3})\sigma_y$. Thus the pressure $p*$ that just causes first yield is $p* = \frac{2}{\sqrt{3}}\frac{t}{r}\sigma_y$.

1.1 and 3 because of work hardening or, in the case of composites, load transfer to the reinforcement.

Cyclic loading can cause a crack to nucleate and grow in a material, culminating in fatigue failure. For many materials there exists a fatigue or *endurance limit*, σ_e (units: MPa), illustrated by the $\Delta\sigma - N_f$ curve of Figure 3.7. It is the stress amplitude $\Delta\sigma$ below which fracture does not occur, or occurs only after a very large number ($N_f > 10^7$) of cycles.

Tensile and compression tests are not always convenient: A large sample is needed and the test destroys it. The hardness test gives an approximate, nondestructive, measure of the strength. The *hardness, H* (SI units: MPa) of a material is measured by pressing a pointed diamond or hardened steel ball into the material's surface (Figure 3.8). The hardness is defined as the indenter force divided by the projected area of the indent. It is related to the quantity we have defined as σ_f by

$$H \approx 3\sigma_f \qquad (3.7)$$

This, in the SI system, has units of MPa. Hardness is commonly reported in a bewildering array of other units, the most common of which is the Vickers H_v scale with units of kg/mm^2. It is related to H in the units used here by

$$H_v = \frac{H}{10}$$

A conversion chart for five hardness scales, relating them to yield strength, appears in Figure 3.9.

The *toughness*, G_{1c} (units: kJ/m^2), and the *fracture toughness*, K_{1c} (units: $MPa/m^{1/2}$ or $MN/m^{1/2}$), measure the resistance of a material to the propagation of a crack. The fracture toughness is measured by loading
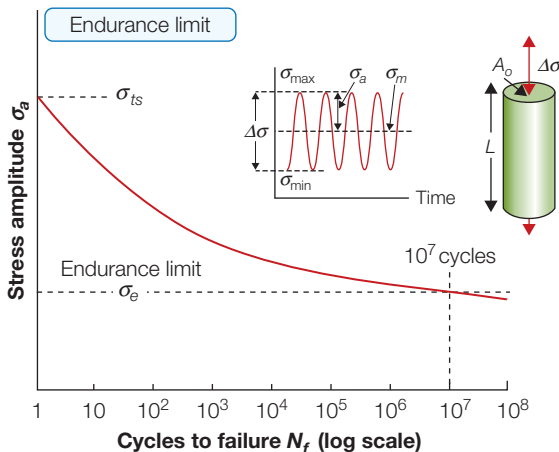
FIGURE 3.7

The endurance limit, σ_e, is the cyclic stress that causes failure in $N_f = 10^7$ cycles.
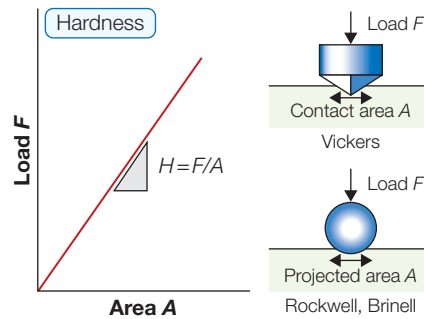
a sample containing a deliberately introduced crack of length $2c$ (Figure 3.10), recording the tensile stress σ^* at which the crack propagates. The quantity K_{1c} is then calculated from

$$K_{1c} = Y\sigma^* \sqrt{\pi c} \qquad (3.8)$$
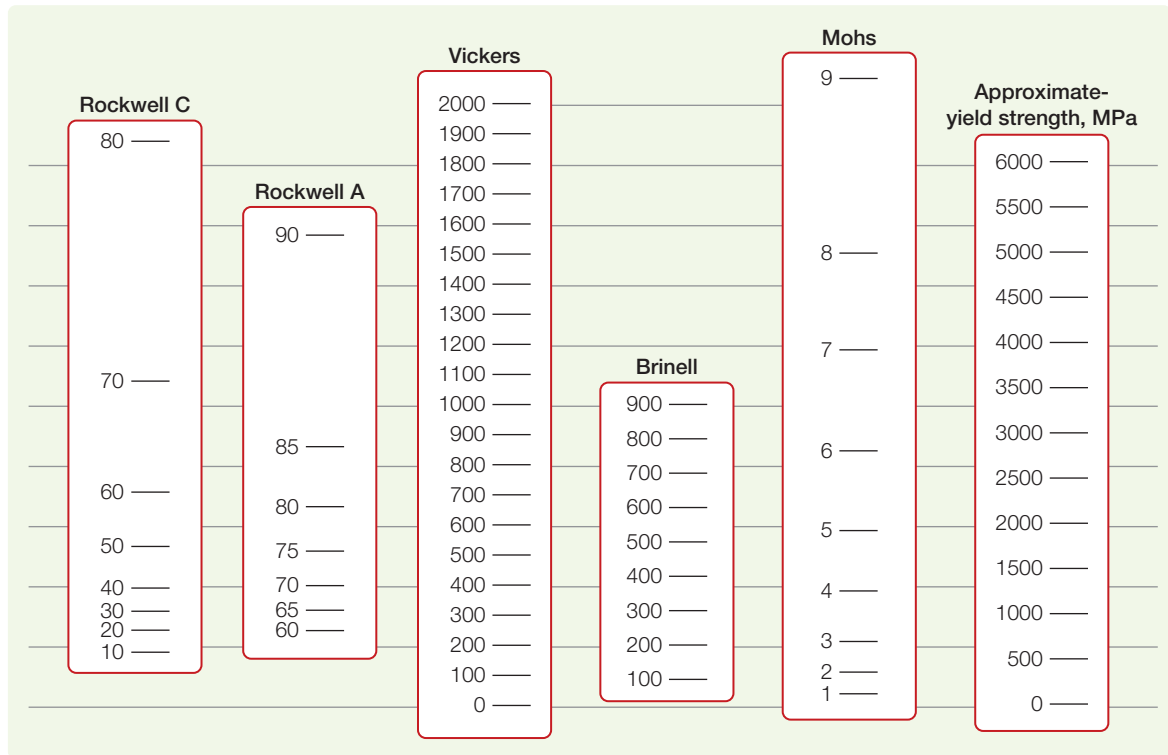
and the toughness from

$$G_{1c} = \frac{K_{1c}^2}{E(1+v)} \qquad (3.9)$$

where Y is a geometric factor, near unity, that depends on details of the

FIGURE 3.8
Hardness is measured as the load, F, divided by the projected area of contact, A, when a diamond-shaped indenter is forced into the surface.
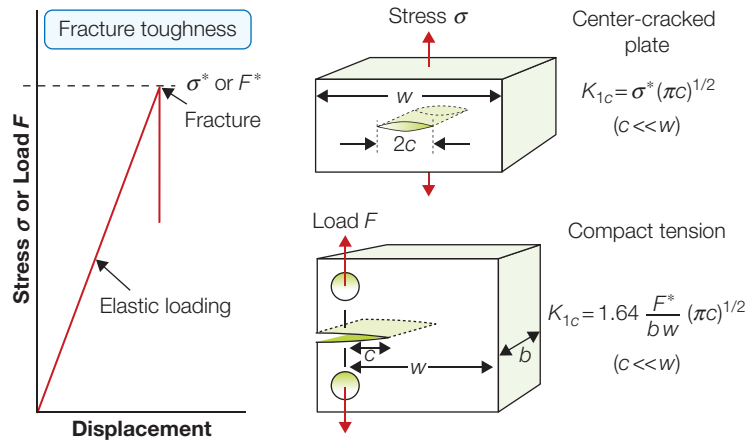

FIGURE 3.9
Commonly used scales of hardness related to each other and to the yield strength.

Strength from hardness

A steel has a hardness of 50 on the Rockwell C scale. Approximately what is its Vickers hardness and yield strength?

Answer
The chart of Figure 3.9 shows that the Vickers hardness corresponding to a Rockwell C value of 50 is approximately $H_v = 500$ and the yield strength is approximately 1,700 MPa.

FIGURE 3.10

The fracture toughness, K_{1c}, measures the resistance to the propagation of a crack. The test specimen containing a crack of length $2c$ fails at stress σ^*. The fracture toughness is then $K_{1c} = Y\sigma^*\sqrt{\pi c}$ where Y is a constant near unity.

sample geometry, E is Young's modulus, and ν is Poisson's ratio. Measured in this way K_{1c} and G_{1c} have well-defined values for brittle materials (ceramics, glasses, and many polymers). In ductile materials a plastic zone develops at the crack tip, introducing new features into the way in which cracks propagate that necessitate more involved characterization. Values for K_{1c} and G_{1c} are, nonetheless, cited, and are useful as a way of ranking materials.

The *loss coefficient, η* (a dimensionless quantity), measures the degree to which a material dissipates vibrational energy (Figure 3.11). If a material is loaded elastically to a stress, σ_{max}, it stores an elastic energy

$$U = \oint_0^{\sigma_{max}} \sigma d\varepsilon \approx \frac{1}{2}\frac{\sigma_{max}^2}{E}$$

Using fracture toughness

A glass floor panel contains micro-cracks up to 2 microns in length. Glass has a fracture toughness of $K_{1c} = 0.6$ MPa.m$^{1/2}$. When the panel is walked upon, stresses as high as 30 MPa appear in it. Is it safe?

Answer

The stress required to make a 2-micron crack (so $c = 10^{-6}$ m) propagate in glass with a fracture toughness of $K_{1c} = 0.6$ MPa.m$^{1/2}$, using Equation 3.8 with $Y = 1$, is

$$\sigma_c = K_{1c}/\sqrt{\pi c} = 339 \text{ MPa}$$

The panel is safe.

per unit volume. If it is loaded and then unloaded, it dissipates an energy:

$$\Delta U = \oint \sigma d\varepsilon$$

The loss coefficient is

$$\eta = \frac{\Delta U}{2\pi U_{\max}} \qquad (3.10)$$

where $U_{\max}$ is the stored elastic energy at peak stress. The value of η usually depends on the time scale or frequency of cycling.

Other measures of damping include the *specific damping capacity*, $D = \Delta U/U$, the *log decrement*, Δ (the log of the ratio of successive amplitudes of natural vibrations), the *phase lag*, δ, between stress and strain, and the "Q" *factor* or *resonance factor*, Q. When damping is small ($\eta < 0.01$) these measures are related by

$$\eta = \frac{D}{2\pi} = \frac{\Delta}{\pi} = \tan\delta = \frac{1}{Q} \qquad (3.11)$$

but when damping is large, they are no longer equivalent.

Wear, the loss of material when surfaces slide against each other, is a multibody problem. Nevertheless, it can, to a degree, be quantified. When solids slide (Figure 3.12), the volume of material lost from one surface, per unit distance slid, is called the wear rate, W (units: m^2). The wear resistance of the
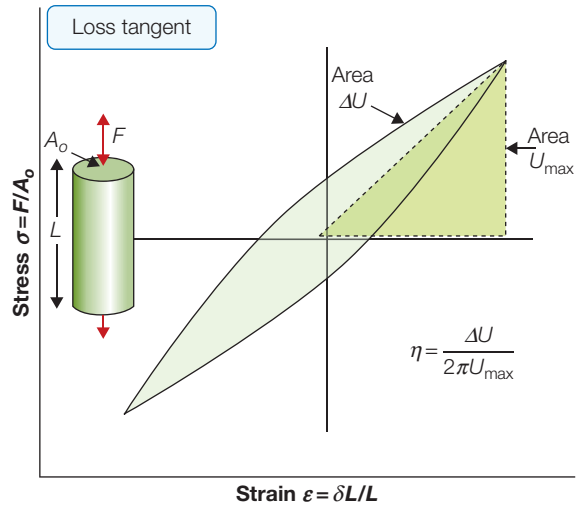
FIGURE 3.11

The loss tangent η measures the fractional energy dissipated in a stress-strain cycle.

Using loss coefficients

A bell with a natural frequency of $f = 1,000$ Hz is made of a material with a loss coefficient of $\eta = 0.01$. For how long will it ring after being struck? If the material is replaced by one of low damping with $\eta = 10^{-4}$ how long will it ring? (Assume the ring has ended when the amplitude of oscillation A has fallen to one-hundredth of its initial value.)

Answer

Let A and $A + dA$ be the amplitudes of the successive cycles (dA is negative). Then

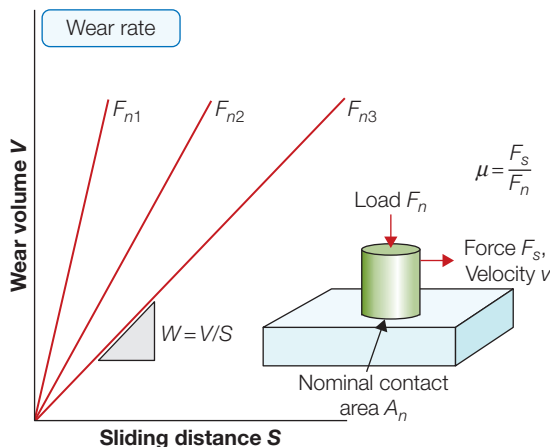$\text{Log} = \left(\frac{A}{A + dA}\right) = \Delta = \pi\eta$ from which

$$\frac{dA}{Ad\eta} = \frac{1}{10^{\pi\eta}} - 1$$

Integrating over n cycles gives $\ln\frac{A}{A_0} = \left(\frac{1}{10^{\pi\eta}} - 1\right)n$ where A_0 is the initial amplitude. When A has fallen to $0.01\,A_0$, the term $\ln(A/A_0) = -4.6$, giving $n = 4.6\left(\frac{10^{\pi\eta}}{10^{\pi\eta} - 1}\right)$. Thus a bell with $\eta = 0.01$ will ring for $n = 66$ cycles, giving a time $n/f = 66$ milliseconds. A bell with $\eta = 10^{-4}$ will ring for $n = 6,400$ cycles and a time of $n/f\ 6.4$ seconds.

surface is characterized by the *Archard wear constant*, K_A (units: 1/MPa) defined by the equation

$$\frac{W}{A} = K_A P \tag{3.12}$$

where A is the area of the slider surface and P is the normal force pressing it onto the other surface. Approximate data for K_A appear in Chapter 4, but must be interpreted as the property of the sliding couple, not of just one member of it.

Thermal properties

Two temperatures, the *melting temperature, T_m*, and the *glass temperature, T_g* (units for both: K or C), are fundamental because they relate directly to the strength of the bonds in the solid. Crystalline solids have a sharp melting point, T_m. Noncrystalline solids do not; the temperature T_g characterizes the transition from true solid to very viscous liquid. It is helpful, in engineering design, to define two further temperatures: the *maximum* and *minimum service temperature*, $T_{\max}$ and $T_{\min}$ (both: K or C). The first tells us the highest temperature at which the material can reasonably be used without oxidation, chemical change,

FIGURE 3.12
Wear is the loss of material from surfaces when they slide. The wear resistance is measured by the Archard wear constant, K_A, defined in the text.

Calculating wear

A steel slider oscillates on a dry steel substrate at frequency $f = 0.2$ Hz and an amplitude $a = 2$ mm under a normal pressure $P = 2$ MPa. The Archard wear constant for steel on steel is $K_A = 3 \times 10^{-8}$ (MPa)$^{-1}$. By how much will the surface of the slider have been reduced in thickness after a time $t = 100$ hours?
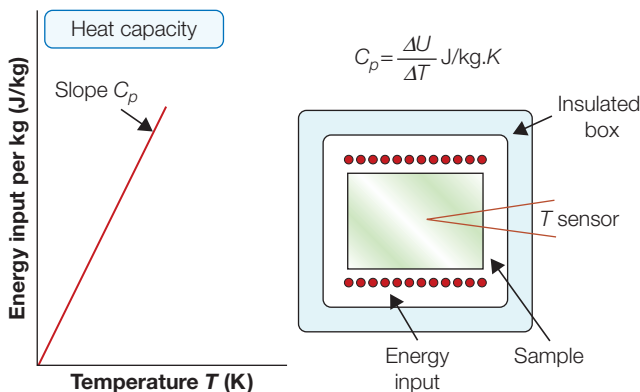
Answer

The distance slid in 100 hours is $d = 4\,a\,f\,t$ m. The thickness x removed from the slider over the time $t = 3.6 \times 10^5$ is

$$x = \frac{Volume\ removed}{Area\ A} = 4\,a\,f\,t\,K_A\,P = 3.5 \times 10^{-5}\text{m} = 36\,\mu\text{m}$$

or excessive creep becoming a problem. The second is the temperature below which the material becomes brittle or otherwise unsafe to use.

It costs energy to heat a material. The *heat capacity* or *specific heat* (units J/kg.K) is the energy to heat 1 kg of a material by 1 K. The measurement is usually made at constant pressure (atmospheric pressure) so it is given the symbol C_p. When dealing with gases, it is more usual to measure the heat capacity at constant volume (symbol C_v), and for gases this differs from C_p. For solids the difference is so slight that it can be ignored, and we shall do so here. The heat capacity is measured by calorimetry (Figure 3.13), which is also the standard way of measuring the glass temperature, T_g. A measured quantity of energy (here, electrical energy) is pumped into a sample of material of known mass. The temperature rise is measured, allowing the energy/kg.K to be calculated. Real calorimeters are more elaborate than this, but the principle is the same.

FIGURE 3.13
The heat capacity—the energy to raise the temperature of 1 kg of material by 1°C.

Using specific heat

How much energy is required to heat a 100 mm cube of copper from room temperature (20°C) to its melting point?

Answer

Data for melting point, T_m, specific heat, C_p, and density, ρ, are listed in Appendix A. The values for copper are $T_m = 1,082°C$, $C_p = 380$ J/kg.K and $\rho = 8,930$ kg/m³. The mass of copper in the cube is $\rho V = 8.93$ kg. The energy to heat it to $\Delta T = 1,062°C$ is

$$\rho V \, C_p \, \Delta T = 3.6 \ \text{MJ}$$

(The energy in a liter of gasoline is 35 MJ.)

The rate at which heat is conducted through a solid at steady state (meaning that the temperature profile does not change with time) is measured by the *thermal conductivity*, λ (units: W/m.K). Figure 3.14 shows how it is measured: by recording the heat flux q (W/m²) flowing through the material from a surface at higher temperature T_1 to a lower one T_2 separated by a distance X. The conductivity is calculated from Fourier's law:

$$q = -\lambda \frac{dT}{dX} = \lambda \frac{(T_1 - T_2)}{X} \tag{3.13}$$

The measurement is not, in practice, easy (particularly for materials with low conductivities), but reliable data are now generally available.
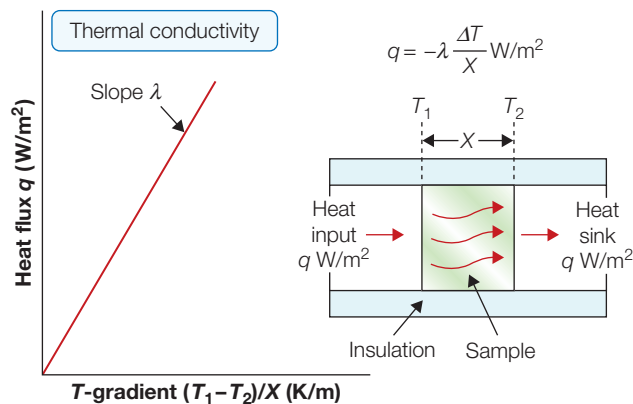
FIGURE 3.14

The thermal conductivity λ measures the flux of heat driven by a temperature gradient dT/dX.

Steady state heat flow

A heat exchanger has an exchange area of $A = 0.5 \text{ m}^2$. It passes heat from a fluid at temperature $T_1 = 100°C$ to a second fluid at $T_2 = 20°C$. The exchange wall is made of copper sheet (thermal conductivity $\lambda = 350$ W/m.K) with a thickness $X = 2$ mm. How much energy flows from one fluid to the other in one hour?

Answer

The temperature gradient $dT/dX = 80/0.002 = 40,000°C$ /m. The total energy Q that passes across the area A over a time $t = 3,600$ seconds is

$$Q = A\,t\,q = A\,t\,\lambda\,\frac{dT}{dX} = 2.5 \times 10^{10}\,\text{J} = 25\,\text{GJ}$$

When heat flow is transient, the flux depends instead on the *thermal diffusivity*, a (units: m^2/s), defined by

$$a = \frac{\lambda}{\rho\,C_p} \tag{3.14}$$

where ρ is the density and C_p is the heat capacity. The thermal diffusivity can be measured directly by measuring the decay of a temperature pulse when a heat source, applied to the material, is switched off; or it can be calculated from λ, via the last equation. The distance x heat diffuses in a time t is approximately

$$x \approx \sqrt{2\,\alpha\,t} \tag{3.15}$$

Most materials expand when they are heated (Figure 3.15). The thermal strain per degree of temperature change is measured by the *linear thermal-expansion coefficient*, α (units: K^{-1} or, more conveniently, "microstrain/°C" or $10^{-6°}C^{-1}$). If the material is thermally isotropic, the volume expansion,

Transient heat flow

You pour boiling water into a tea-glass with a wall thickness $x = 3$ mm. How many seconds have you got to carry it to the table before it becomes to hot to hold? (The thermal conductivity of glass is $\lambda = 1.1$ W/m.K, its density is $\rho = 2,450 \text{ kg/m}^3$ and its heat capacity $C_p = 800$ J/kg.K.)

Answer

Inserting the data into Equation (3.14) gives a thermal diffusivity for glass of $a = 5.6 \times 10^{-7} \text{ m}^2/\text{s}$. Inserting this into Equation (3.13) gives the approximate time

$$t \approx \frac{x^2}{2a} = 8\,\text{seconds}$$
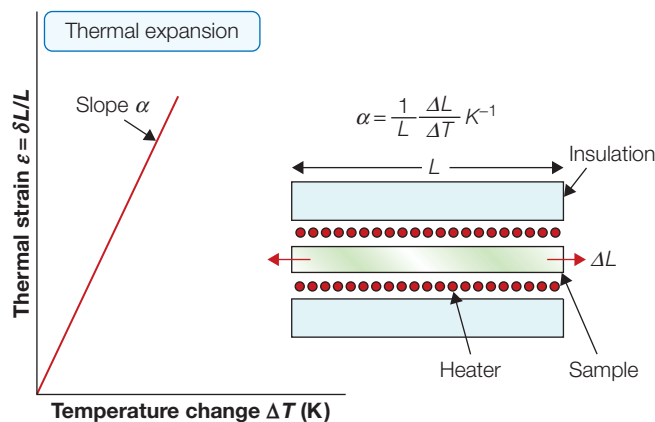
FIGURE 3.15
The linear-thermal expansion coefficient α measures the change in length, per unit length, when the sample is heated.

Thermal stress

An aluminum pipe is rigidly clamped to the face of a concrete building. On a hot day the face of the building in direct sun rises to 80°C, and because the expansion of the aluminum is greater than that of concrete, stress appears in it. What is the value of the stress if the original clamping was done on a day when the temperature was 20°C?

Answer
The expansion coefficient of aluminum is $\alpha = 22.5 \times 10^{-6}/°C$, that of concrete is $\alpha = 9 \times 10^{-6}/°C$, using means of the ranges in Appendix A. The aluminum pipe is rigidly clamped, so the difference in thermal strain $\Delta \alpha \Delta T = 13.5 \times 10^{-6} \times 60 = 8.1 \times 10^{-4}$. This has to be accommodated by elastic compression of the aluminum (modulus $E = 75$ GPa from Appendix A), giving a stress $\Delta \alpha \Delta T E = 61$ MPa. This is enough to cause a soft aluminum to yield.

per degree, is 3α. If it is anisotropic, two or more coefficients are required, and the volume expansion becomes the sum of the principal thermal strains.

The *thermal shock resistance* ΔT_s (units: K or C) is the maximum temperature difference through which a material can be quenched suddenly without damage. I, and the *creep resistance* are important in high-temperature design. Creep is the slow, time-dependent deformation that occurs when materials are loaded above about $\frac{1}{3}T_m$ or $\frac{2}{3}T_g$. Design against creep is a specialized subject. Here we rely instead on avoiding the use of a material above its maximum service temperature, T_{max}, or , for polymers, the "heat deflection temperature."

Electrical properties

The *electrical resistivity*, ρ_e (SI units Ω.m or, commonly, $\mu\Omega$.cm), is the resistance of a unit cube with unit potential difference between a pair of its faces (Figure 3.16). It has an immense range, from a little more than 10^{-8} in units of Ω.m for good conductors (equivalent to 1 $\mu\Omega$.cm) to more than 10^{16} Ω.m (10^{24} $\mu\Omega$.cm) for the best insulators. The electrical conductivity, κ_e (units Siemens per meter, S/m or $(\Omega.m)^{-1}$), is simply the reciprocal of the resistivity.
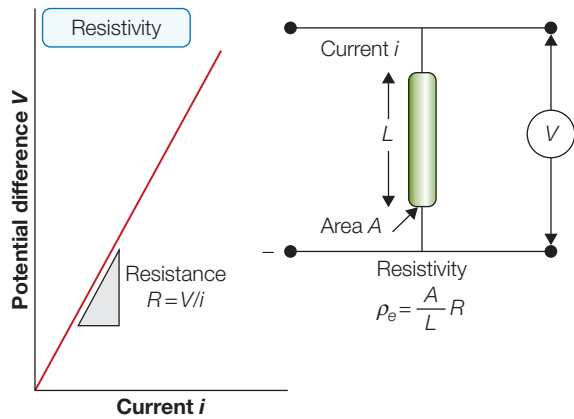
When an insulator (or *dielectric*) is placed in an electric field, it becomes polarized and charges appear on its surfaces that tend to screen the interior from the electric field. The tendency to polarize is measured by the *dielectric constant*, ε_r, a dimensionless quantity (Figure 3.17). Its value for free space and, for practical purposes, for most gasses, is 1. Most insulators have values between 2 and 30, though low-density foams approach the value 1 because they are largely air.

FIGURE 3.16

Electrical resistivity, ρ_e, is measured as the potential gradient, V/L, divided by the current density, i/A. It is related to resistance, R, by $\rho_e = AR/L$.

What does ε_r measure? Two conducting plates separated by a dielectric make a capacitor. Capacitors store charge. The charge, Q (units: coulombs), is directly proportional to the voltage difference between the plates, V (volts):

$$Q = CV \tag{3.16}$$

where C (farads) is the capacitance. The capacitance of a parallel plate capacitor of area A, separated by empty space (or by air), is

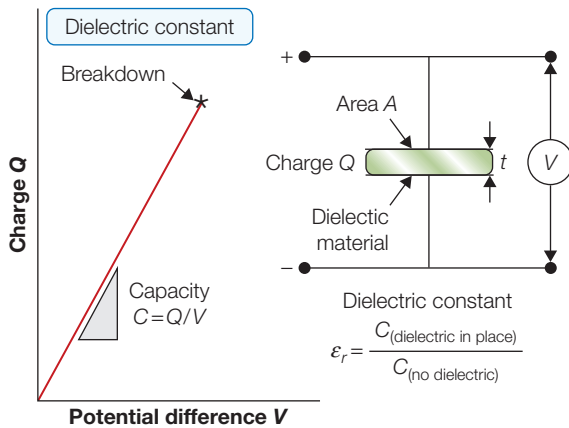$$C = \varepsilon_o \frac{A}{t} \tag{3.17}$$

Resistivity and resistance

Tungsten has a conductivity of $\kappa_e = 8.3 \times 10^6$ Siemens. What is the resistance of a tungsten wire of radius $r = 100$ microns in diameter and length $L = 1$ m?

Answer

The resistivity of tungsten $\rho_e = 1/\kappa_e = 1.2 \times 10^{-7}$ Ω.m. The resistance R of the wire is

$$R = \rho_e \frac{L}{\pi r^2} = 3.8\,\Omega$$

FIGURE 3.17

Dielectric constant: a measure of the ability of an insulator to polarize.

where ε_0 is the *permittivity of free* space (8.85×10^{-12} *F/m*, where *F* is farads). If the empty space is replaced by a dielectric, capacitance increases because of its polarization. The field created by the polarization opposes the field *E*, reducing the voltage difference *V* needed to support the charge. Thus the capacity of the condenser is increased to the new value:

$$C = \varepsilon \frac{A}{t} \tag{3.18}$$

where ε is the *permittivity of the dielectric* with the same units as ε_0. It is usual to cite not this but the *relative permittivity* or *dielectric constant*, ε_r:

$$\varepsilon_r = \frac{C_{with\ dielectric}}{C_{no\ dielectric}} = \frac{\varepsilon}{\varepsilon_o} \tag{3.19}$$

making the capacitance

$$C = \varepsilon_r \varepsilon_o \frac{A}{t} \tag{3.20}$$

When charged, the energy stored in a capacitor is

$$\frac{1}{2} Q V = \frac{1}{2} C V^2 \tag{3.21}$$

and this can be large: "Super-capacitors" with capacitances measured in farads store enough energy to power a hybrid car.

Polarization involves the small displacement of charge (of either electrons or ions) or of molecules that carry a dipole moment when an electric field is applied to the material. An oscillating field drives the charge between

Stray capacitance

The time constant τ for charging or discharging a capacitor is

$$\tau = R C$$

where *R* is the resistance of the circuit. That means that stray capacitance in an electronic circuit (capacitance between neighboring conducting lines or components) slows its response. What material choices minimize this?

Answer

Choosing materials with low resistivity ρ_e for the conductors (to minimize *R*) and choosing insulators with low dielectric constant ε_r to separate them (to minimize *C*), minimizes τ.

two alternative configurations. This charge motion is like an electric current that—if there were no losses—would be 90° out of phase with the voltage. In real dielectrics this current dissipates energy, just as a current in a resistor does, giving it a small phase shift, δ (Figure 3.18). The *loss tangent, tan δ*, also called the *dissipation factor, D*, is the tangent of the loss angle. The *power factor, P_f*, is the sine of the loss angle. When δ is small, as it is for the materials of interest here, all three are essentially equivalent:

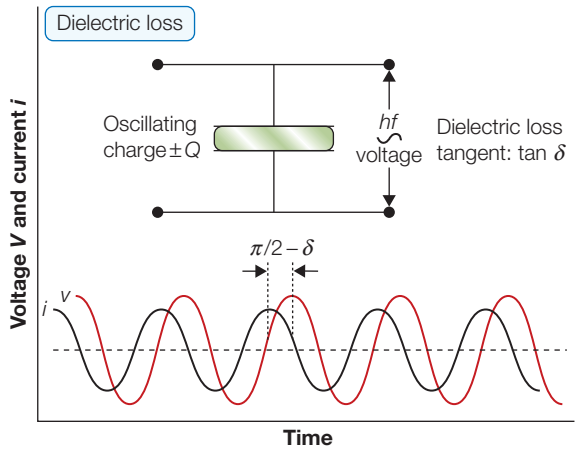$$P_f \approx D \approx \tan\delta \approx \sin\delta \qquad (3.22)$$

More useful, for our purposes, is the *loss factor L*, which is the loss tangent times the dielectric constant:

$$L = \varepsilon_r \tan\delta \qquad (3.23)$$

FIGURE 3.18

Dielectric loss, important in dielectric heating, as explained in the text.

It measures the energy dissipated by a dielectric when in an oscillating field. If you want to select materials to minimize or maximize dielectric loss, then the measurement you want is L.

When a dielectric material is placed in a cyclic electric field of amplitude E and frequency f, power P is dissipated and the field is correspondingly attenuated. The power dissipated per unit volume (W/m³) is

$$P \approx f E^2 \varepsilon \tan\delta = f E^2 \varepsilon_o \varepsilon_r \tan\delta = f E^2 \varepsilon_o L \qquad (3.24)$$

Dielectric heating

A nylon component is placed in a microwave cavity with a field strength $E = 10^4$ V/m and a frequency $f = 10^{10}$ Hz for a time $t = 100$ s. The dielectric loss factor for nylon is $L = 0.1$, its density is $\rho = 1130$ kg/m³ and its heat capacity is $C_p = 1650$ J/kg.K. Assuming no heat loss, how hot will the component become?

Answer

The heat generated by the field is $Q = Pt = f E^2 \varepsilon_0 L t = 8.85 \times 10^7$ J/m³. The heat capacity of nylon per unit volume is $C_p\rho = 1.86 \times 10^6$ J/m³.K. The temperature rise, ΔT is

$$\Delta T = \frac{f E^2 \varepsilon_o L t}{C_p\rho} = 47.6\,°C$$

where, as before, ε_r is the dielectric constant of the material and *tan δ* is its loss tangent. This power appears as heat; the higher the frequency or the field strength and the greater the loss factor $L = \varepsilon_r \tan \delta$, the greater the heating and energy loss. Sometimes this dielectric loss is exploited in processing—for example, in radio frequency welding of polymers.

The *breakdown potential* (units: MV/m) is the electrical potential gradient at which an insulator breaks down and a damaging surge of current flows through it. It is measured by increasing, at a uniform rate, a 60-Hz alternating potential applied across the faces of a plate of the material until breakdown occurs, typically at a potential gradient between 1 and 100 million volts per meter (units: MV/m).

Optical properties

All materials allow for some passage of light, although for metals it is exceedingly small. The speed of light when in the material, v, is always less than that in vacuum, c. A consequence is that a beam of light striking the surface of such a material at an angle of incidence, α, enters the material at an angle β, the angle of refraction. The *refractive index*, n (dimensionless), is

$$n = \frac{c}{v} = \frac{\sin \alpha}{\sin \beta} \tag{3.25}$$

It is related to the dielectric constant, ε_r, at the same frequency by

$$n \approx \sqrt{\varepsilon_r}$$

The refractive index depends on wavelength and thus on the color of the light. The denser the material, and the higher its dielectric constant, the greater the refractive index. When $n = 1$, the entire incident intensity enters the material, but when $n > 1$, some is reflected. If the surface is smooth and polished, it is reflected as a beam; if rough, it is scattered. The percentage reflected, R, is related to the refractive index by

$$R = \left(\frac{n-1}{n+1}\right)^2 \times 100 \tag{3.26}$$

As n increases, the value of R approaches 100%.

Eco-properties

The *embodied energy* (units MJ/kg) is the energy required to extract 1 kg of a material from its ores and feedstock. The associated CO_2 *footprint* (units: kg/kg) is the mass of carbon dioxide released into the atmosphere during the production of 1 kg of material. These and other eco-attributes are the subject of Chapter 15.