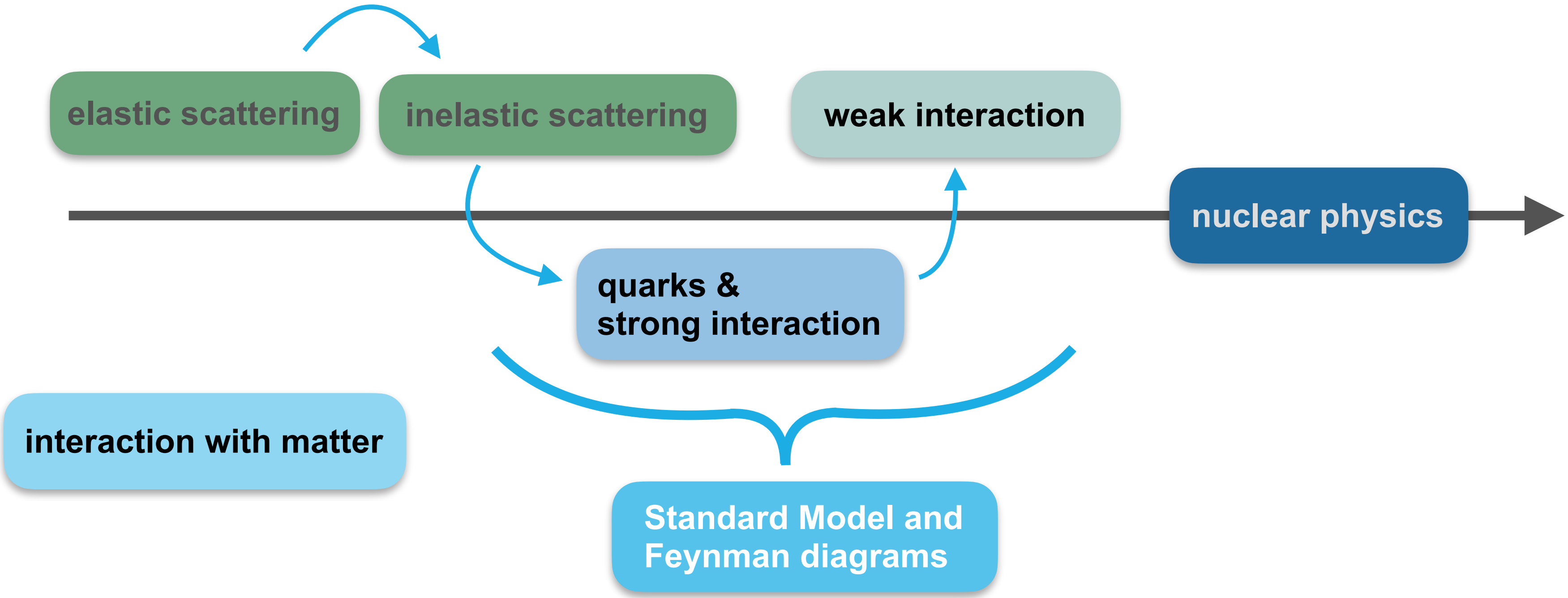
Introduction to Nuclear and Particle Physics

Lesson 14

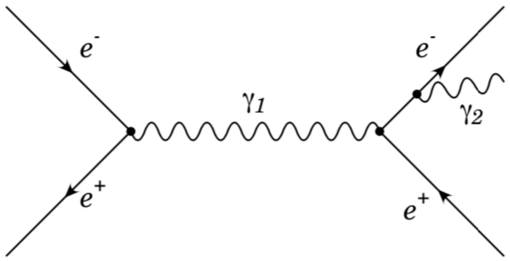
the big recap game


elastic scattering
inelastic scattering
weak interaction
nuclear physics
quarks &
strong interaction
interaction with matter
Standard Model and
Feynman diagrams

Today: Recap of selected topics / problems

Relativistic kinematics

Virtual Photon



Which mass and momentum does γ_1 have?
(Expressed in CMS of $e^+ - e^-$ pair, natural units)

A) $m_\gamma = E_e$ $\quad \left|\vec{p_\gamma}\right| = E_e$

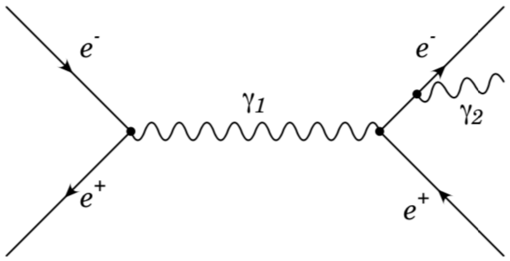
B) $m_\gamma = E_e$ $\quad \left|\vec{p_\gamma}\right| = 0$

C) $m_\gamma = 2 \cdot E_e$ $\quad \left|\vec{p_\gamma}\right| = 0$

D) $m_\gamma = 0$ $\quad \left|\vec{p_\gamma}\right| = E_e$

E) $m_\gamma = 0$ $\quad \left|\vec{p_\gamma}\right| = 2 \cdot E_e$

Virtual Photon



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4-momentum in CMS:

$$\mathbf{P} = \mathbf{P}_{e+} + \mathbf{P}_{e-} = \begin{pmatrix} E_{e^+} + E_{e^-} \\ \vec{p}_{e^+} + \vec{p}_{e^-} \end{pmatrix} = \begin{pmatrix} 2E_e \\ 0 \end{pmatrix}$$

Conservation of 4-momentum:

$$\mathbf{P}_{\text{gamma}} = \mathbf{P}$$

$$\sqrt{s} = \sqrt{P_\mu P^\mu}$$

$$\Rightarrow \quad \vec{p}_\gamma = 0 \qquad m_\gamma = \sqrt{s} = 2E_e$$

Mass, energy and momentum in special relativity

Relativistic particle

Total energy

$$E = \sqrt{m^2c^4 + p^2c^2}$$

$$E = \gamma \cdot mc^2$$

Momentum

$$p = \gamma \cdot mv$$

rest energy

$$E_0 = mc^2$$

"Equivalence of mass and energy"

kinetic energy

$$E_{kin} = E - E_0$$

Non-relativistic particle

$$E_{kin} \ll mc^2$$

Total energy

$$E = mc^2 + \frac{p^2}{2m}$$

Momentum

$$p = mv$$

Relativistic factors

$$\beta = \frac{v}{c}$$

$$\beta = \frac{pc}{E}$$

$$\beta = 0\ldots1 \qquad \gamma = 1\ldots\infty$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\gamma = \frac{E}{mc^2}$$

Excited nucleus - group work

A nucleus has a mass m and an excited state ΔE above its ground state which can be reached by absorbing a γ-ray:

$$\gamma + A \to A^* \,. \tag{2}$$

It is assumed that $\Delta E/c^2$ is not so small compared to the mass m of the nucleus A.

b) **(4 points)** Find the energy of the photon E_γ needed to resonantly excite the nucleus to the excited state assuming that the nucleus prior to the absorption is at rest.

Hints: Use momentum conservation: $\left|\vec{p}_\gamma\right| = \left|\vec{p}_{A*}\right|$

Use $m_{A*} = m_A + \frac{\Delta E}{c^2}$ and $E_\gamma = \left|\vec{p}_\gamma\right|$

You can use natural units!

Deploy energy conservation and simplify

Excited nucleus - group work - solution

A nucleus has a mass m and an excited state ΔE above its ground state which can be reached by absorbing a γ-ray:

$$\gamma + A \rightarrow A^* \ . \qquad (2)$$

It is assumed that $\Delta E/c^2$ is not so small compared to the mass m of the nucleus A.

b) **(4 points)** Find the energy of the photon E_γ needed to resonantly excite the nucleus to the excited state assuming that the nucleus prior to the absorption is at rest.

Energy conservation: → $\left|\vec{p}_\gamma\right| = \left|\vec{p}_{A*}\right|$ → $E_\gamma = \left|\vec{p}_\gamma\right|$

$$E_0 = m_A + p_\gamma = E^* = \sqrt{m_{A*}^2 + p_{A*}^2}$$

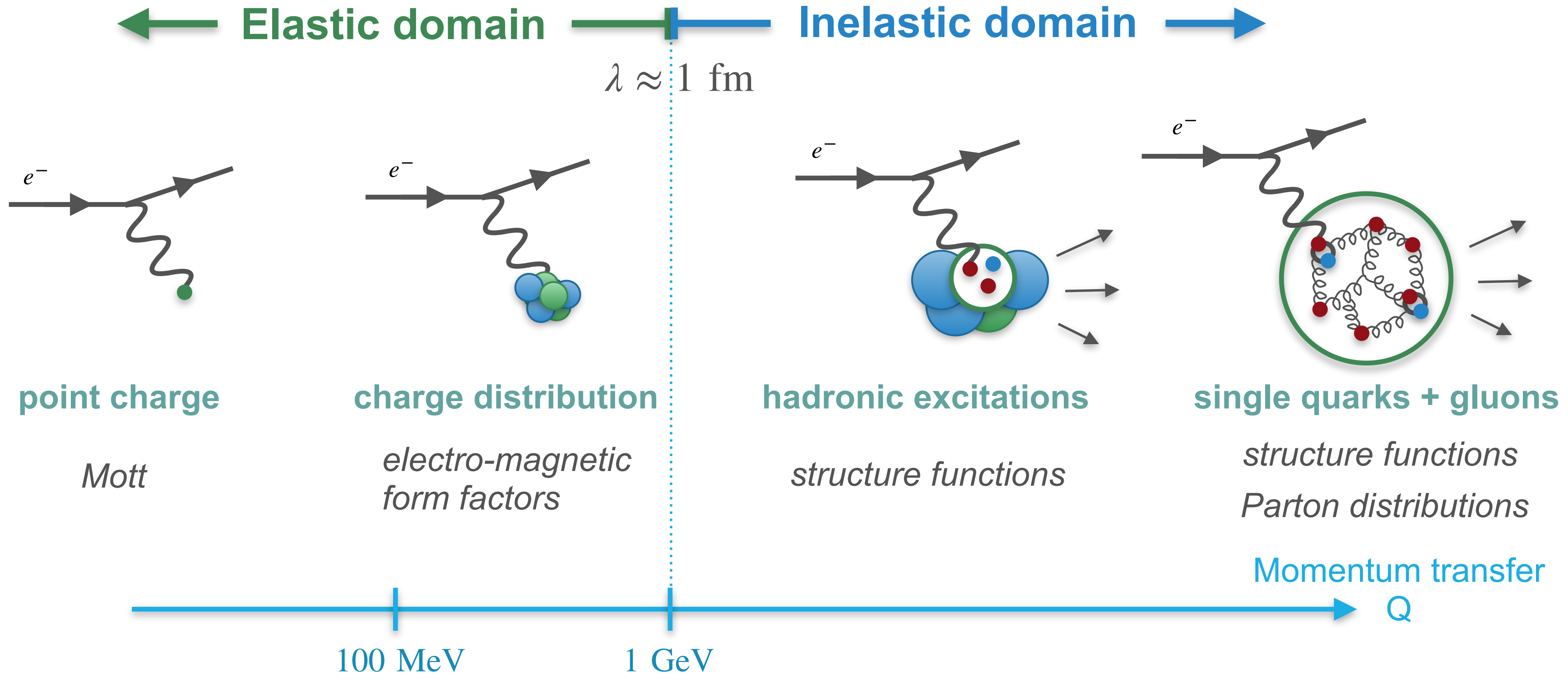
$$m_A + p_\gamma = \sqrt{m_{A*}^2 + p_\gamma^2}$$

$$\left(m_A + p_\gamma\right)^2 = \left(m_A + \Delta E\right)^2 + p_\gamma^2$$

$$\ldots \qquad E_\gamma = p_\gamma = \Delta E + \frac{\Delta E^2}{2m_A}$$

Scattering

Electron scattering and structure determination



Hint: Many are correct here!

Which statements about electron-proton scattering are correct?

- ☐ Rutherford scattering includes the effect of spin interaction between the projectile particle and the target nucleus.
- ☐ Mott scattering ignores the finite size of the nucleus.
- ☐ Mott scattering is based on relativistic kinematic.
- ☐ The Rosenbluth cross section (Rosenbluth formula) is based on relativistic kinematic, and accounts for recoil (Rückstoss) and finite size of the nucleous.
- ☐ Deep inelastic scattering proves that the proton contains various partons.
- ☐ *The question and/or one or many answers seem to be ambiguous. Reason:*

. . . .

. . . .

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Game: skribbl

Proposed words: next slide

invariant mass
rest mass
neutrino mass
muon decay
bending radius
virtual particle
lifetime
beta-minus decay
beta-plus decay
cross-section
form factor
annihilation
photoeffect
compton scattering
pair production
bethe-bloch equation
virtual photon

quark
vertex
feynman diagram
four-momentum
coupling constant
parity
parity violation
weak charged current
neutron decay
bremsstrahlung
rutherford experiment
resonance
hadron
clebsch-gordan coefficients
isospin
orbital momentum
spin
hadron jet
collider
bragg peak
helicity

quark mixing
cabibbo angle
lepton
weak interaction
strong interaction
CKM matrix
left-handed
parton
fission
fusion
neutrino oscillations
nuclear chart
magic numbers
binding energy
reactor
shell model
critical mass
moderator
kaon oscillations
pion decay

Energy loss in matter

Overview: Energy loss of e/m particles in matter

Photons

$$I(x) = I_0 \cdot e^{-\mu x}$$

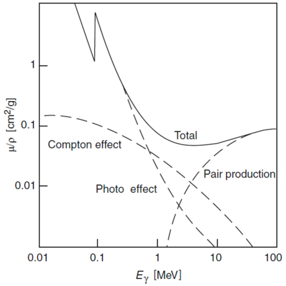
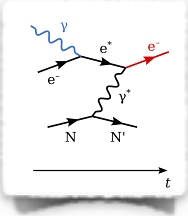
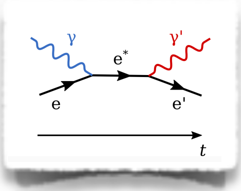


Photo effect



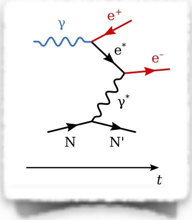
$$\mu_{PE} \sim \frac{Z^n}{E_\gamma^{3.5}}$$

Compton effect



$$\mu_{PE} \sim \frac{Z}{E}$$

Pair production



$$\mu_{PC} \sim Z^2 E_\gamma$$

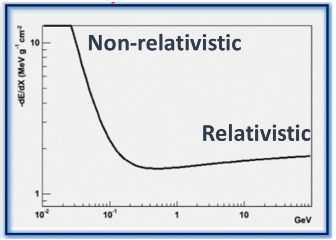
Additionally: elastic photon scattering

Charged particles

$$\frac{dE}{dx} = \left(\frac{dE}{dx}\right)_{\text{col}} + \left(\frac{dE}{dx}\right)_{\text{rad}}$$

ionization losses

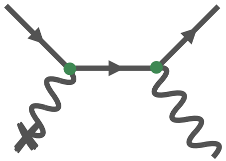
Bethe-Bloch



all particles similar
only depends on v

radiative losses

$$\left(\frac{dE}{dx}\right)_{rad} \sim \frac{Z^2 E}{m^2}$$



- important for e^+/e^-
- heavier particles: only ultra-relativistic

A proton, a $^2H^+$ ion and a $^3He^{++}$ ion of the same initial kinetic energy $E = 10$ MeV pass through the same medium where they lose energy by collision with the atomic electrons, as described by the Bethe-Bloch formula. What is the relation between energy losses?

- ☐ $\left(\frac{dE}{dx}\right)_{^3He^{++}} > \left(\frac{dE}{dx}\right)_{^2H^+} > \left(\frac{dE}{dx}\right)_p$
- ☐ $\left(\frac{dE}{dx}\right)_{^3He^{++}} < \left(\frac{dE}{dx}\right)_{^2H^+} < \left(\frac{dE}{dx}\right)_p$
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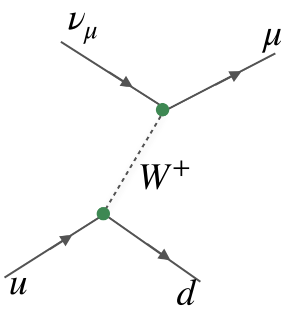
Bethe-Bloch losses depend only on velocity of particle, not the mass.
At 10 MeV (non-relativistic!), the proton is faster than the heavier nuclei.

Feynman diagrams

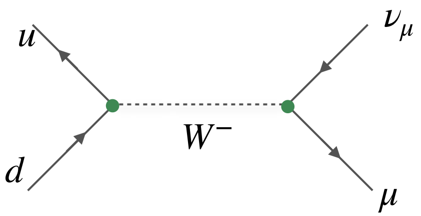
Weak diagrams

Which of the following Feynman diagrams are valid?

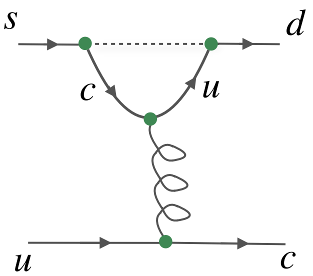
A)



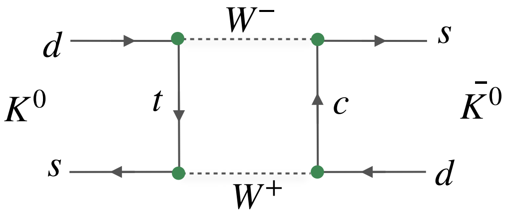
C)



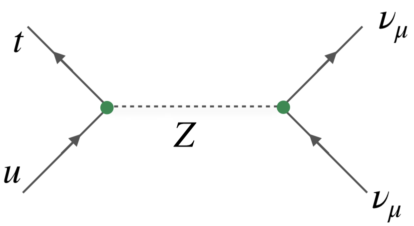
E)



B)



D)



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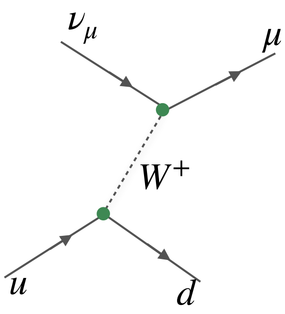
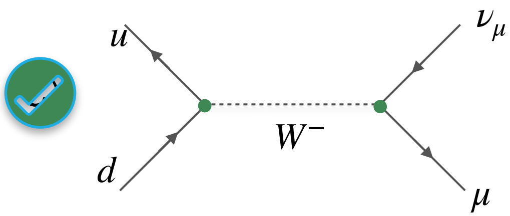
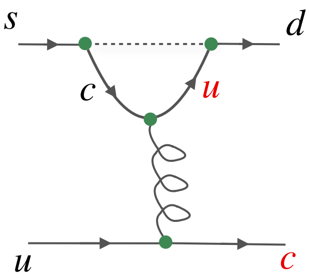


Wrong charge of W

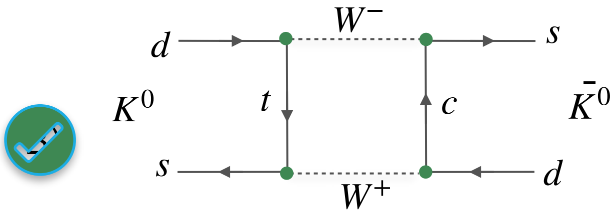


π^- decay

E)

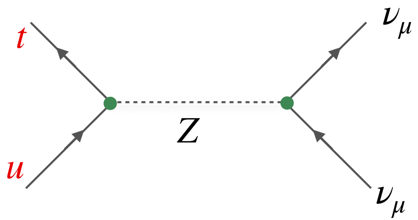


strong interaction
does not
change flavour!



Kaon oscillation

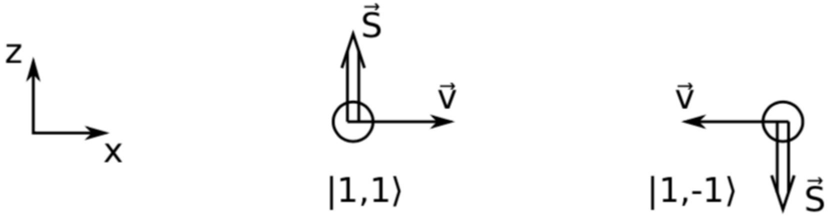
D)



Z boson does not mix quark generations!

Clebsch-Gordan Coefficients

Two nuclei with same mass, both with spin 1, move in $\pm x$ direction, respectively. The relative orbital angular momentum of the two nuclei is zero. They collide to form a single nucleus. What is the probability that the formed nucleus will have a total angular momentum $|J, J_z>=|1, 0>$ when the two colliding nuclei are polarized in $+z$ and $-z$ as shown in the figure below.



1×1

m_1	m_2	$J=2$, $M=+2$
+1	+1	1

m_1	m_2	$J=2$, $M=+1$	$J=1$, $M=+1$
+1	0	1/2	1/2
0	+1	1/2	−1/2

m_1	m_2	$J=2$, $M=0$	$J=1$, $M=0$	$J=0$, $M=0$
+1	−1	1/6	1/2	1/3
0	0	2/3	0	−1/3
−1	+1	1/6	−1/2	1/3

m_1	m_2	$J=2$, $M=-1$	$J=1$, $M=-1$
0	−1	1/2	1/2
−1	0	1/2	−1/2

m_1	m_2	$J=2$, $M=-2$
−1	−1	1

- ☐ $1/2$
- ☐ $\sqrt{2}$
- ☐ $\sqrt{1/2}$
- ☐ $\sqrt{2/3}$
- ☐ $\sqrt{3/2}$

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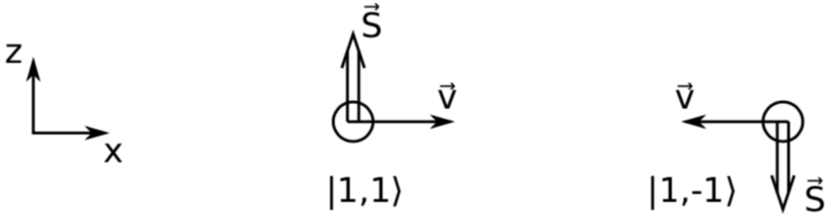

1×1

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m_1	m_2	$J=2$, $M=+1$	$J=1$, $M=+1$
+1	0	1/2	1/2
0	+1	1/2	−1/2

m_1	m_2	$J=2$, $M=0$	$J=1$, $M=0$	$J=0$, $M=0$
+1	−1	1/6	1/2	1/3
0	0	2/3	0	−1/3
−1	+1	1/6	−1/2	1/3

m_1	m_2	$J=2$, $M=-1$	$J=1$, $M=-1$
0	−1	1/2	1/2
−1	0	1/2	−1/2

m_1	m_2	$J=2$, $M=-2$
−1	−1	1

- [x] $1/2$
- [ ] $\sqrt{2}$
- [ ] $\sqrt{1/2}$
- [ ] $\sqrt{2/3}$
- [ ] $\sqrt{3/2}$

Standard model

Particles and their interactions

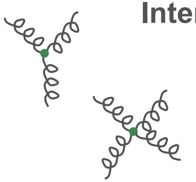
fermions ↓ / bosons →		electro-magnetic	weak		strong
		γ	$W^{\pm}$	Z	g
quarks	u c t d s b	X	X	X	X
charged leptons	e μ τ	X	X	X	
neutral leptons	ν_e ν_μ ν_τ		X	X	

The sectors of the Standard Model

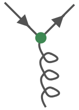
QCD sector
color g_s

strong **gauge bosons:** 8 gluons

Interactions:



gluon - gluon
quark - gluon
(and more)



Standard Model

Electroweak sector
Hypercharge
Weak isospin

weak + electromagnetic g_w Qe

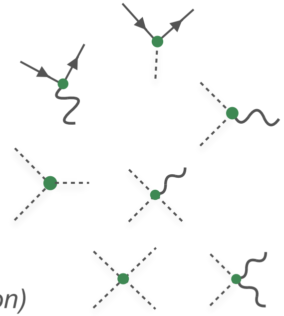
gauge bosons: $W^{\pm}, Z, \gamma$

Interactions:

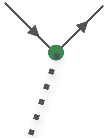
bosons - fermions
bosons - bosons
(and more)

Special for weak component:

- *Quark mixing (CKM)*
- *Chiral theory (Parity violation)*



Yukawa sector
m_f



mass terms for fermions (not for ν)

Interactions:

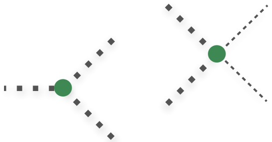
Higgs - fermions

Higgs sector
"Symmetry breaking" m_W m_Z m_h

mass terms for Higgs, W, Z

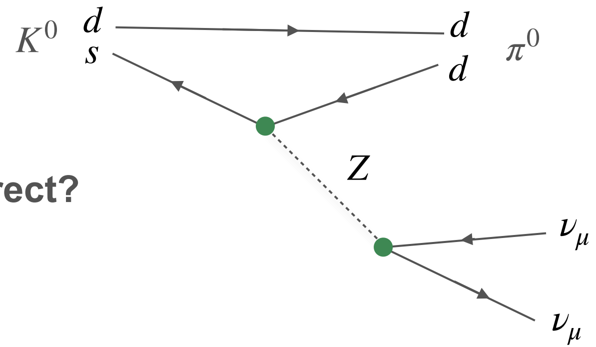
Interactions:

Higgs - W
Higgs - Z
Higgs - Higgs
(and more)



Weak currents

Which statement about the diagrams on the right is correct?

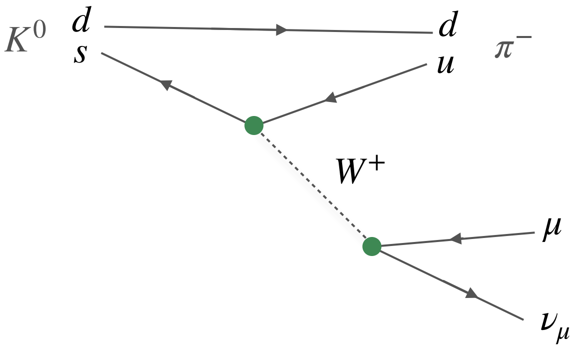


A) Both processes are valid.

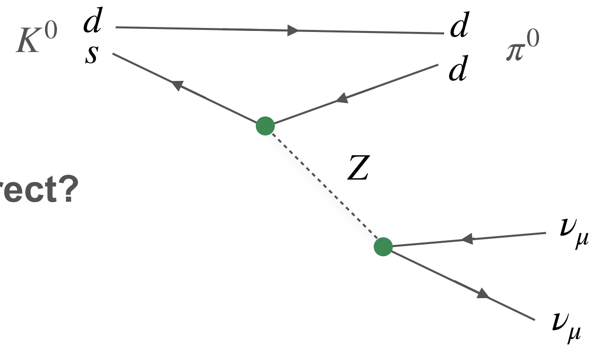
B) The processes are both not valid because the strangeness is not conserved.

C) Quark mixing is part only of the W interaction. The upper diagram is not valid.

D) The charge is not conserved in the lower process. Only the upper diagram is valid.



Weak currents



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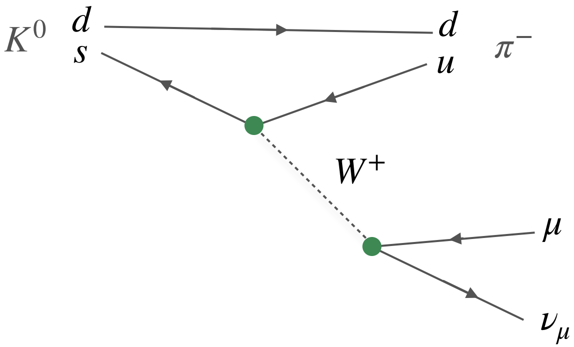

The Standard Model

Which statements about the standard model are correct?

A) The Higgs boson can interact with all particles of the SM.

B) The CP symmetry is slightly violated in the weak interaction.

C) Parity violation can be observed, but is not part of the Standard Model.

D) The top quark couples stronger to the Higgs than all other quarks.

E) The fermion masses are a direct consequence of the symmetry breaking in the SM.

The Standard Model

Which statements about the standard model are correct?

A) The Higgs boson can interact with all particles of the SM.

not gluons, photons, neutrino

✓ The CP symmetry is slightly violated in the weak interaction.

Maximal parity violation is implemented in the SM.

C) Parity violation can be observed, but is not part of the Standard Model.

✓ The top quark couples stronger to the Higgs than all other quarks.

The fermion masses arise from the Yukawa term.

E) The fermion masses are a direct consequence of the symmetry breaking in the SM.