# Introduction to Nuclear and Particle Physics

## Lesson 7

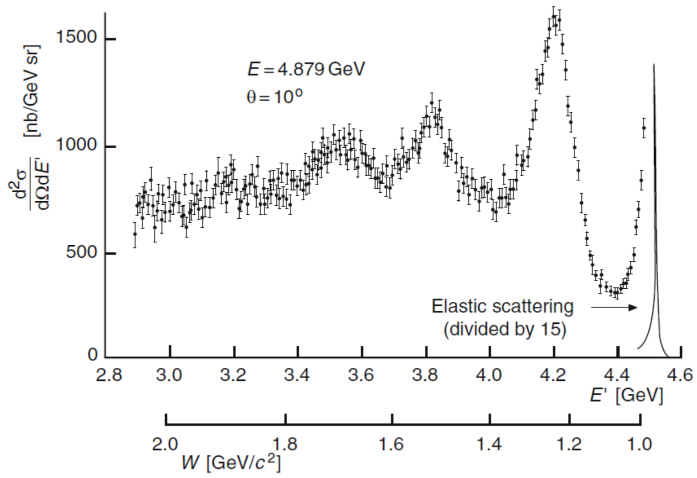
*inelastic scattering*

# Warm-up question 1

In a scattering experiment, electrons are scattered of protons.

The plot on the right shows hadronic resonances for electron scattering on protons.

**Which statements are correct?**




A) Hadronic resonances decay via the strong interaction.

B) The life-time cannot be determined because it is too short

C) If a resonance is produced, the electron must have transferred energy to the proton.

D) The electron interacted strongly with the nucleon.

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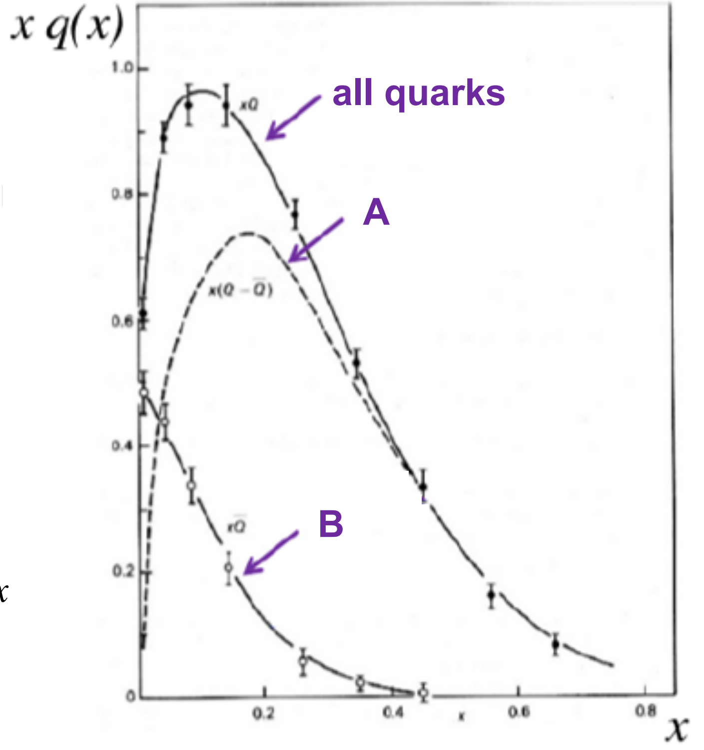



A) Curve A corresponds to the momentum fraction carried by sea quarks.

B) The total momentum fraction of the proton which a specific quark q carries can be calculated by

$$f_q = \int_0^1 x q(x)\,\mathrm{d}x$$

C) Gluons are not included in the plot because they do not contribute to the proton’s momentum.

D) It is unlikely to find a valence quark that carries more than 50% of the proton momentum.

# The plan for today

hadronic
resonances

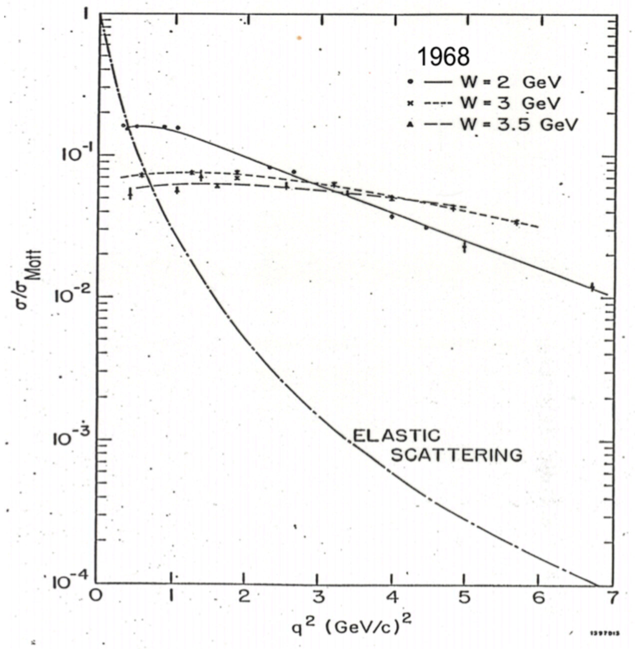
Inelastic scattering

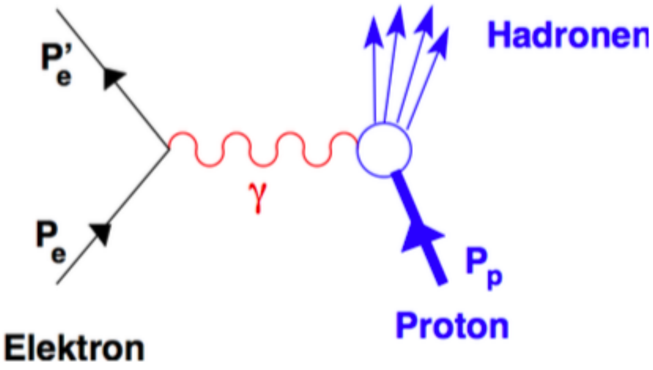
Bjorken scaling

structure functions

Parton model

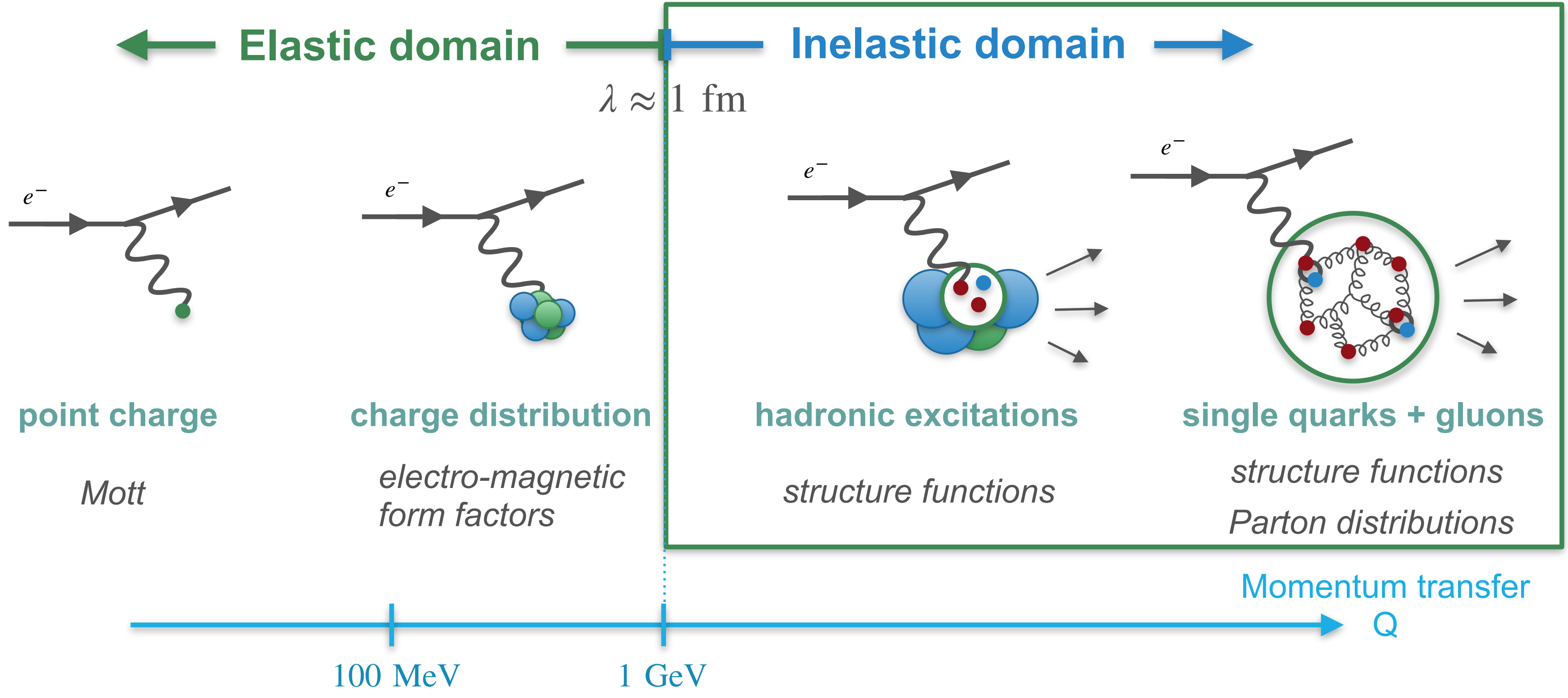
# Entering the inelastic regime



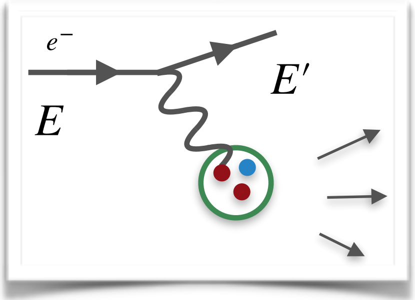

[ Breidenbach, et.al., Phys. Rev. Let. 23 (1969) 935. ]

# Electron scattering and structure determination

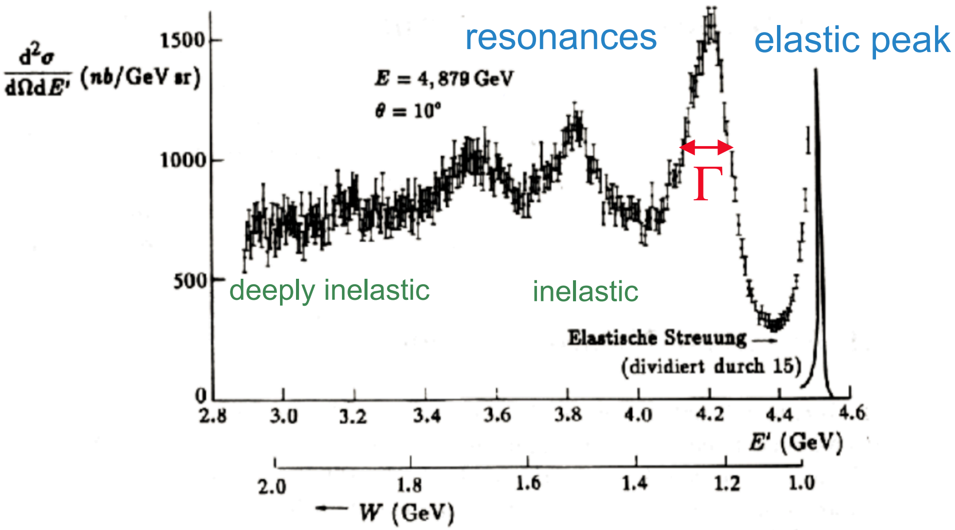


# Hadronic resonances

= excited states of nucleons which decay via strong interaction. These resonances occur at specific momentum transfer (i.e. invariant mass).




look at energy lost by $e^-$ !



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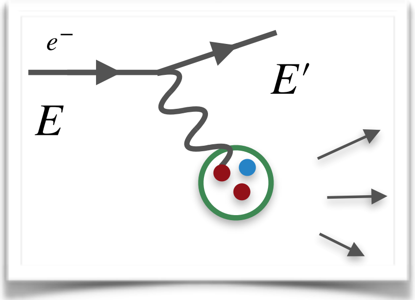


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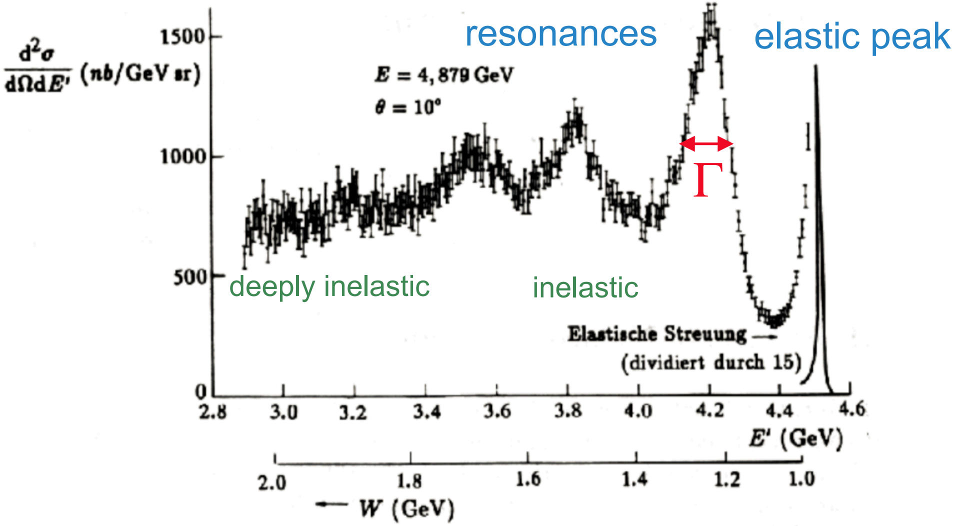


**Example:** $\Delta$ resonances

$W = 1232$ MeV

e.g. $(p^+)^* \rightarrow \Delta^+ \rightarrow p^+ + \pi^0$

$(n)^* \rightarrow \Delta^0 \rightarrow p^+ + \pi^-$

**Even higher energy transfer: nucleon is completely destroyed (deeply inelastic)**

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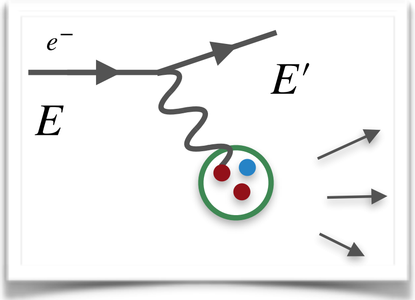
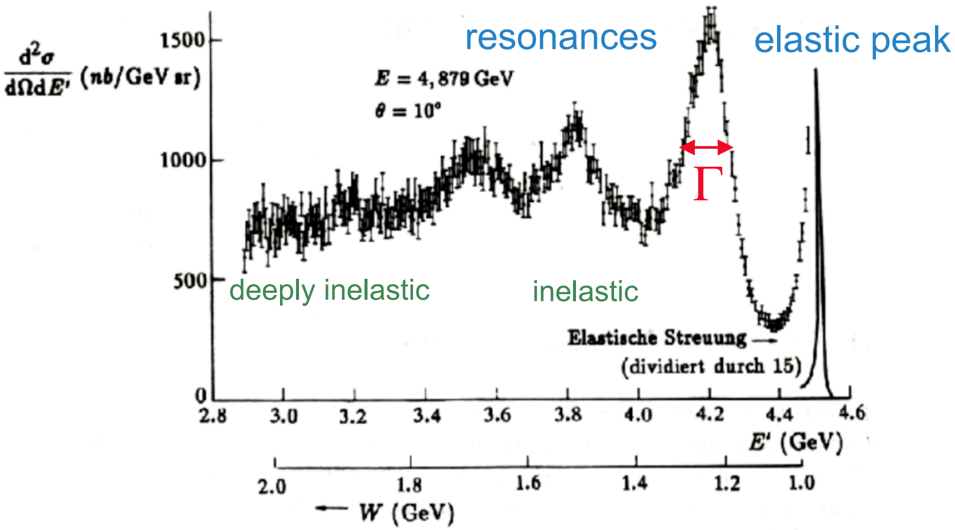



look at energy lost by e[-] !



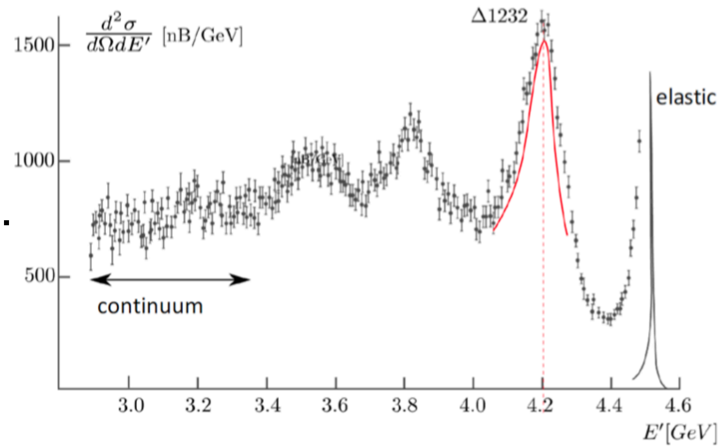

## Width and lifetime of resonance

Uncertainty principle $\Delta E \cdot \Delta t \approx \hbar$

$\Rightarrow$ lifetime: $\tau \approx \frac{\hbar}{\Gamma}$

*Example:* $\Delta(1232)$ *lives only* $5.5 \cdot 10^{-24}$ *s !*

# Hadronic resonances question




**Consider this figure describing the differential cross section of e-p scattering in a fixed target experiment. The measurement was performed at a fixed angle and a fixed initial energy of the electron.**

## Which statements are correct?

A) The produced $\Delta$ resonance is a $\Delta^{++}$ resonance.

B) The produced $\Delta$ resonance is a $\Delta^0$ resonance because the exchanged photon is neutral.

C) The energy $E'$ in the above figure is the energy of the exchanged photon.

D) The initial electron energy is larger than 4.5 GeV

E) The energy of the elastically scattered electron at the measurement angle is 4.5 GeV

F) The electrons that have produced a $\Delta$ resonance have a kinetic energy of about 0.3 GeV less than elastically scattered electrons.

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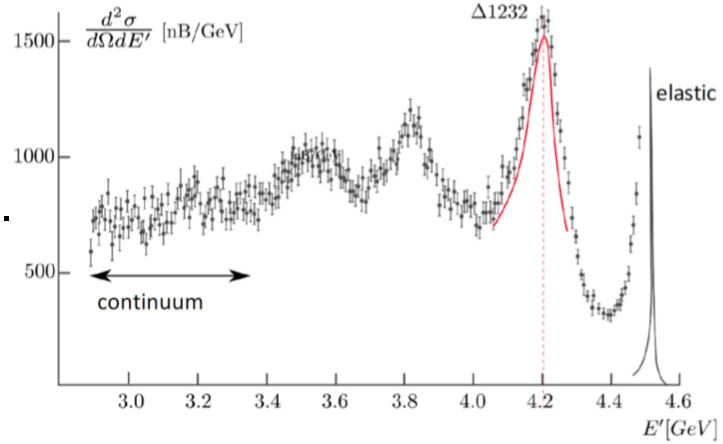


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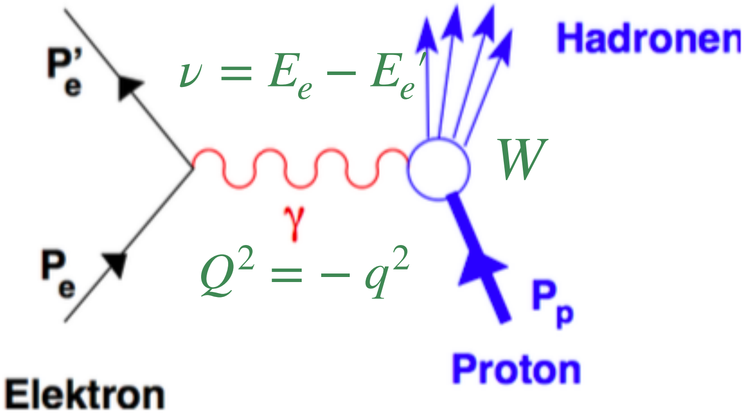
# Bjorken scale and structure functions




| | |
|---|---|
| $W$ | invariant mass of hadron |
| $\nu$ | energy transfer |
| $q^2$ | 4-momentum squared of photon |

**Bjorken scale variable** $x = \frac{Q^2}{2P_p \cdot q}$

elastic $x = 1$

inelastic $x < 1$ $\quad x = \frac{Q^2}{Q^2 + W^2 - m_p^2}$

*how inelastic is the process?*

**Structure functions**

$$\left(\frac{d^2\sigma}{d\Omega dE'}\right)_{\text{ep-inel.}} = \left(\frac{d\sigma}{d\Omega}\right)^*_{\text{Mott}} \left\{W_2(Q^2,\nu) + 2W_1(Q^2,\nu)\tan^2\left(\frac{\Theta}{2}\right)\right\}$$

$\Rightarrow$ Analogue to form factors

**Question 25** (*2 Points*) The quarks ...

A) ...interact only through the strong interaction.

B) ...interact only through the strong and the weak interactions.

C) ...interact strongly through the exchange of gluons.

D) ...interact electromagnetically through the exchange of photons.

E) ...interact only through the strong and the electromagnetic interactions but not through the weak interaction.

F) ...interact through all interactions including the gravitational interaction.

None of the above expressions are correct.

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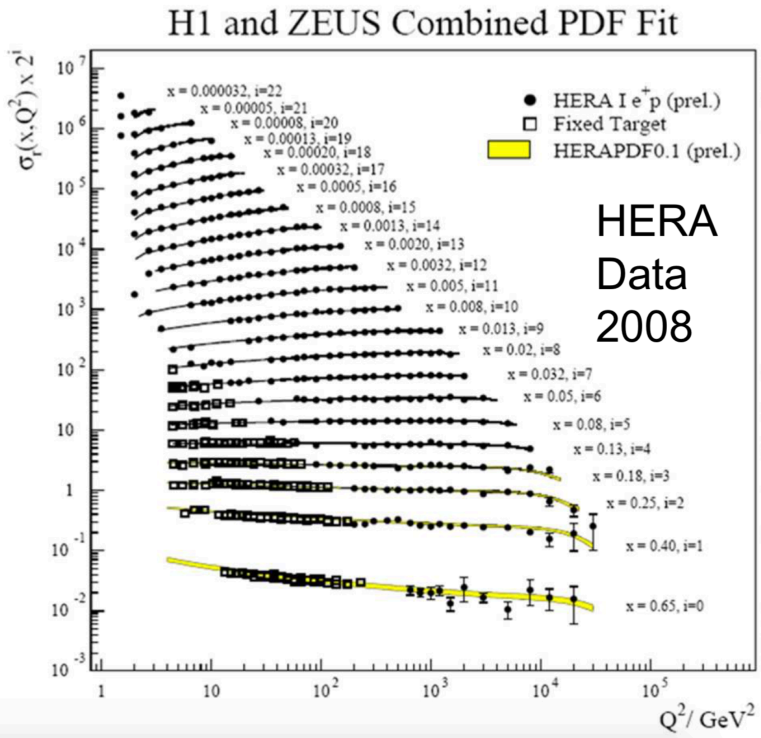
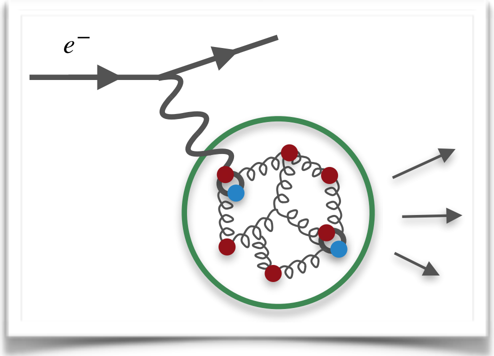
- ☐ ...interact only through the strong and the electromagnetic interactions but not through the weak interaction.
- ■ ...interact through all interactions including the gravitational interaction.
- ☐ None of the above expressions are correct.

**But how did people learn about quarks?**

# Bjorken scaling and Parton model




## Bjorken scaling:

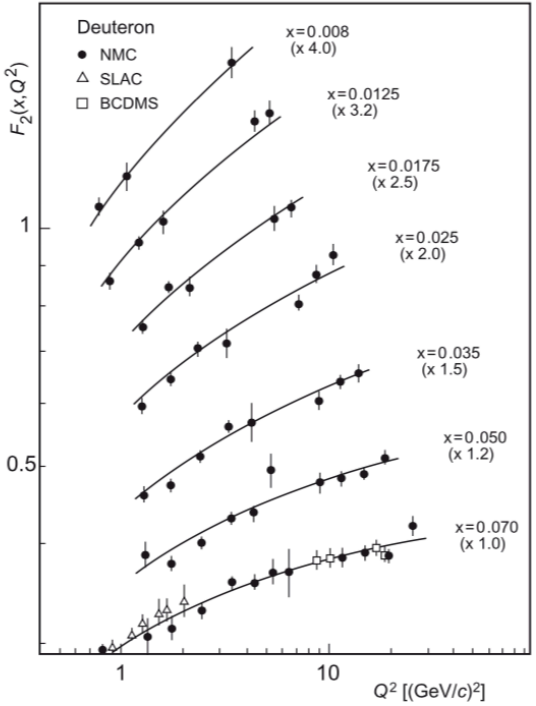
Structure functions nearly independent of $Q^2$

## What do we learn from this?

Nucleon consists of point-like particles: Partons!

$x$ : momentum fraction of nucleon carried by parton

# Scale violation for small x

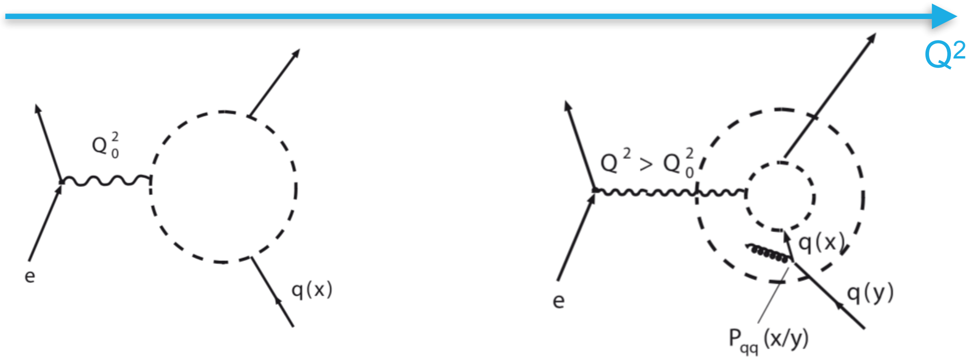



Bjorken scaling breaks down for very small x.

## How can we explain this?

We dive deeper into the loops of the strong interaction.

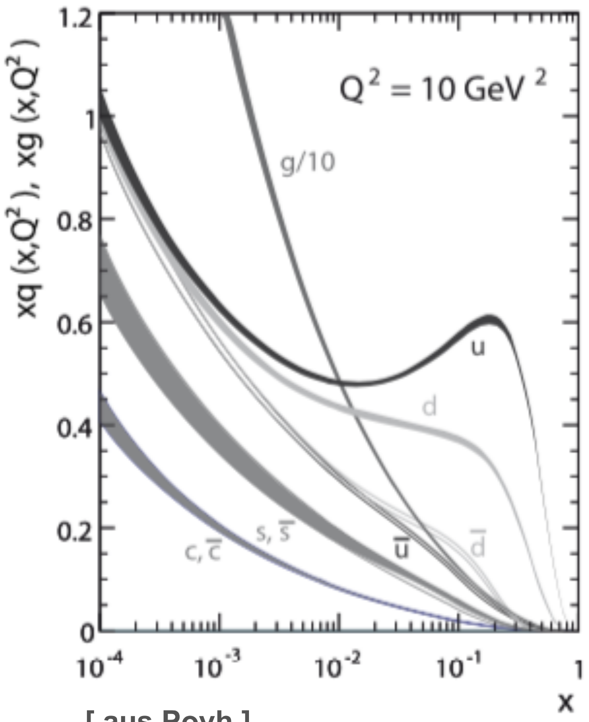
⇒ sea quarks and gluons (small momentum fractions)






ENGAGING PHYSICS TUTORING EPT

# Parton distribution functions




[ aus Povh ]

**Which partons do we find at which momentum fraction?**

*Experimentally determined!*

$q_i\ \mathrm{d}x$:

**probability to hit quark of type i with** $x\ \epsilon\ [x, x + dx]$

*Note for small x:*

- *gluons are dominant*
- *valence quarks loose importance*

# Physics girl!

https://www.youtube.com/watch?v=LraNu_78sCw

## Group work I - Exam questions on quarks and gluons

Discuss the following exam questions in the group and decide what is the correct answer.

Take your time to explain your ideas and approaches to each other.

Please be ready to discuss your results and ideas later in the plenary.

**Question 4** (*2 Points*) Consider the strong process

$$K^-(\bar{u}s) + p(uud) \to K^+(u\bar{s}) + X.$$

What is the quark content of the X particle?

- sdd.
- ddd.
- $s\bar{s}d$.
- ssd.
- $s\bar{s}u$.
- $u\bar{s}\bar{d}$.
- $\bar{u}\bar{u}\bar{d}$.

In electron-proton deep inelastic scattering, the observation of Bjorken scaling invariance gave evidence that the electron scatter-off of point-like constituents. For fixed $x$ they observed that ...

... the structure functions $F_{1,2}(x, Q^2)$ decrease linearly with decreasing $Q^2$.

... the structure functions $F_{1,2}(x, Q^2)$ increase with decreasing $Q^2$.

... the structure functions $F_{1,2}(x, Q^2)$ are roughly constant versus $Q^2$.

... the structure functions $F_{1,2}(x, Q^2)$ oscillate with $Q^2$.

... the structure functions $F_{1,2}(x, Q^2)$ have an exponentially decay in an oscillating fashion.

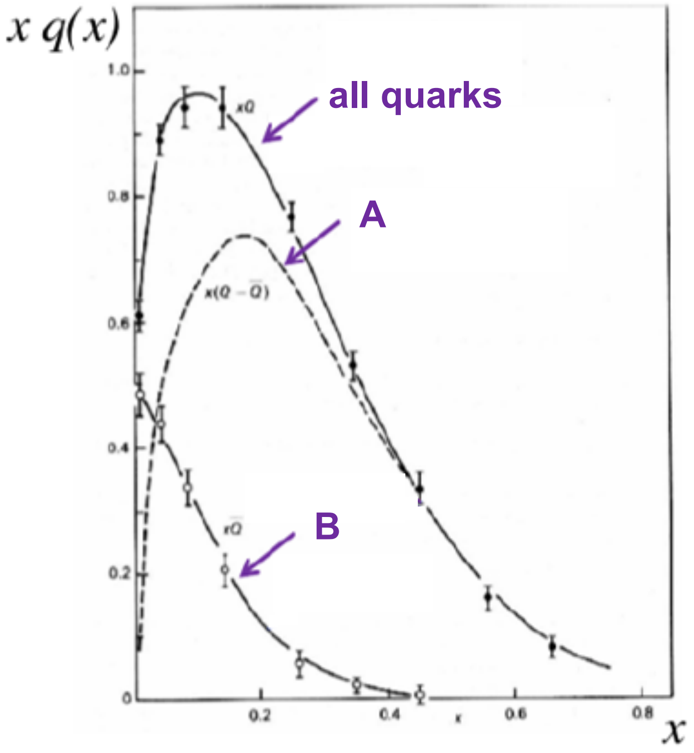
... the structure functions $F_{1,2}(x, Q^2)$ are delta-functions.

**Question 25 Fraction of momentum carried by quarks and gluons (*4 Points*)**

$q(x)$ and $\bar{q}(x)$ are the probabilities to find a quark and an anti-quark, respectively, with fractional momentum of the proton between $x$ and $x + dx$.

a) **(2 points)** How can you compute the fraction of momentum carried by all quarks and anti-quarks? Assume that $q(x)$ and $\bar{q}(x)$ are known.

b) **(2 points)** Use the figure below to make a rough estimate of the fraction of the proton momentum carried by the gluons.



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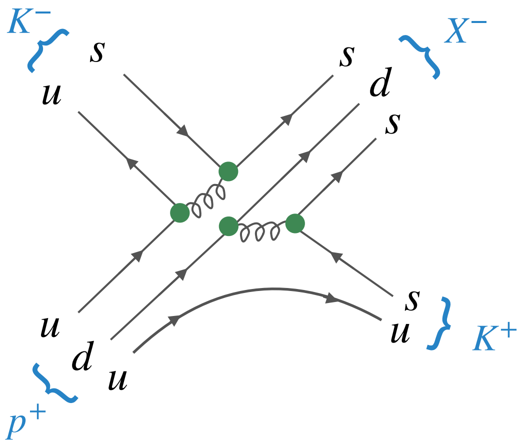
example Feynman diagram:




Only valence quarks considered here!

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**Question 25** **Fraction of momentum carried by quarks and gluons (*4 Points*)**

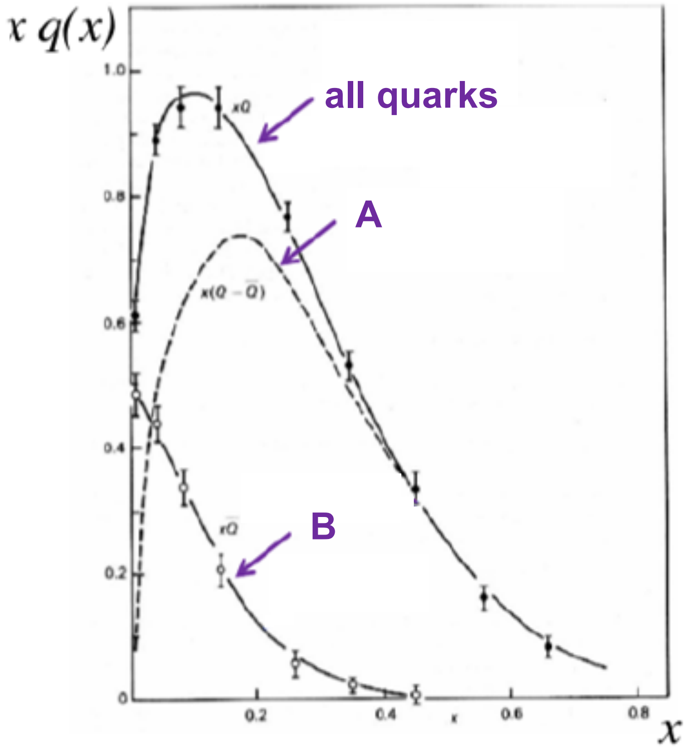
$q(x)$ and $\bar{q}(x)$ are the probabilities to find a quark and an anti-quark, respectively with a fractional momentum of the proton between $x$ and $x + dx$.

a) **(2 points)** The fraction $f$ of momentum carried by the quarks and anti-quarks can be calculated simply by the integral

$$f = \int_0^1 dx\,[x\bar{q}(x) + xq(x)]. \tag{1}$$

b) **(2 points)** Use the figure below to make a rough estimate of the fraction of the proton momentum carried by the gluons.

$$f_g = 1 - f_{q/\bar{q}} = 1 - \int_0^1 \mathrm{d}x\,(qx + \bar{q}x) \approx 0.5$$

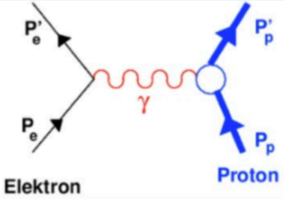
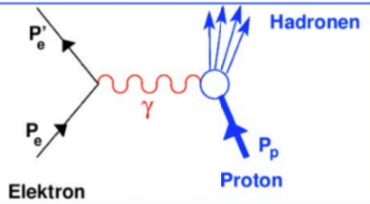



roughly 50 % of the proton's momentum is carried by gluons!

# Summary

## Elastic vs inelastic

(taken from script)

| Elastische Streuung | Tiefinelastische Streuung |
|---|---|
|   |   |
| Dynamik durch Formfaktoren $(G_M, G_E)$ beschrieben | Dynamik durch Strukturfunktionen $(W_1, W_2)$ beschrieben |
| Bei vorgegebener $E_e$ 1 freier Parameter: $\sigma = f(\theta)$ | Bei vorgegebener $E_e$ 2 freie Parameter: $\sigma = f(E', \theta)$ oder $f(Q^2, \nu)$ |
| $W = M: 2M\nu - Q^2 = 0$ | $W > M: 2M\nu - Q^2 > 0$ |
| Wirkungsquerschnitt $\left(\frac{d^2\sigma}{d\Omega dE'}\right)_{\text{ep-el.}} = \left(\frac{d\sigma}{d\Omega}\right)_R \delta\left(\nu - \frac{Q^2}{2M_p}\right) \cdot \left\{A(Q^2)\cos^2\left(\frac{\Theta}{2}\right) + B(Q^2)\sin^2\left(\frac{\Theta}{2}\right)\right\}$ | Wirkungsquerschnitt $\left(\frac{d^2\sigma}{d\Omega dE'}\right)_{\text{ep-inel.}} = \left(\frac{d\sigma}{d\Omega}\right)_R \cdot \left\{W_2(Q^2,\nu)\cos^2\left(\frac{\Theta}{2}\right) + 2W_1(Q^2,\nu)\sin^2\left(\frac{\Theta}{2}\right)\right\}$ |

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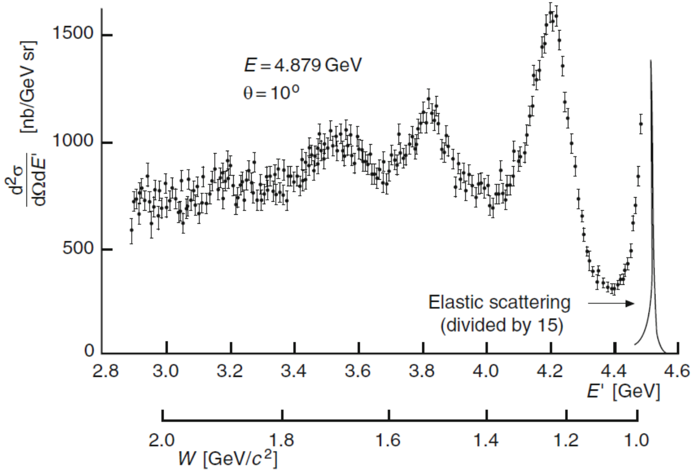


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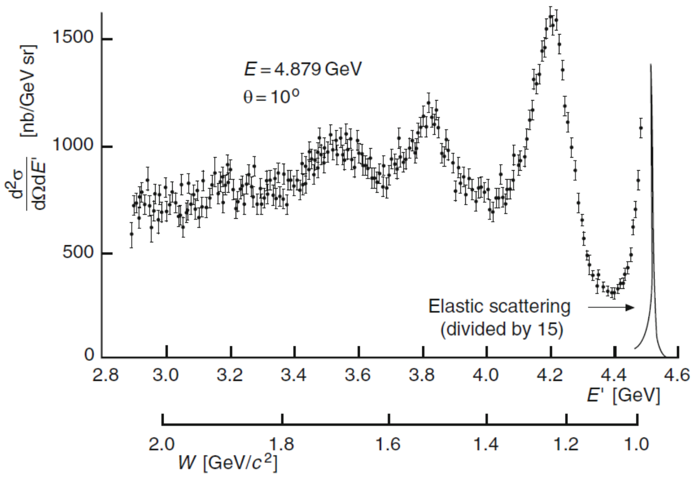


**Which statements are correct?**

A) ✓ Hadronic resonances decay via the strong interaction.

B) The life-time cannot be determined because it is too short

Is short, but can be calculated from width $\Gamma$

C) ✓ If a resonance is produced, the electron must have transferred energy to the proton.

D) The electron interacted strongly with the nucleon.

electromagnetically!
Electrons cannot do strong

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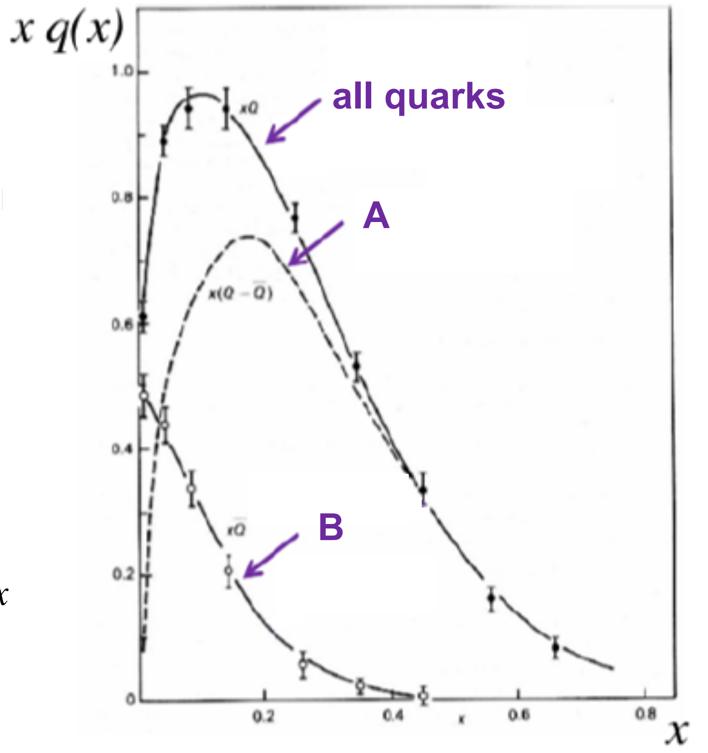


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$$f_q = \int_0^1 x q(x)\, \mathrm{d}x$$

C) Gluons are not included in the plot because they do not contribute to the proton's momentum.

Gluons carry ~ 50% of the momentum! Dominant at low x.

✓ It is unlikely to find a valence quark that carries more than 50% of the proton momentum.