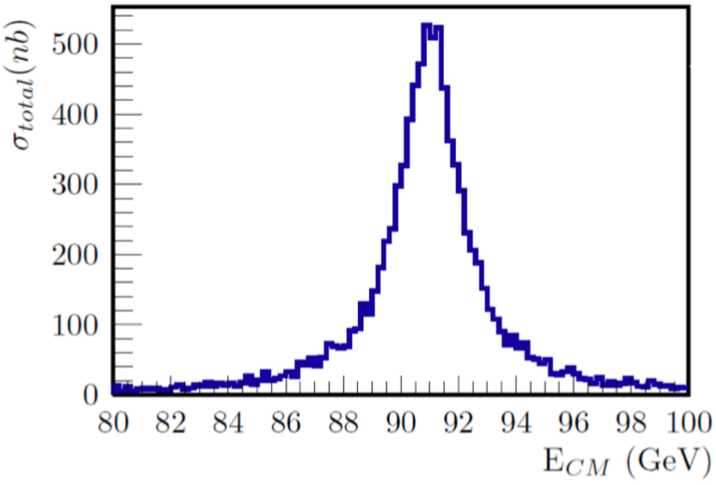
Introduction to Nuclear and Particle Physics

Lesson 3

introduction to Feynman diagrams

Warmup question



In a collider experiment, the scattering of
An e^+-e^- pair into muons is observed
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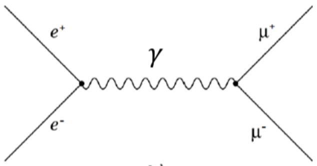
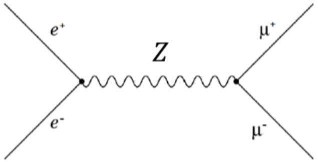
Plotting the total cross section against the energy, one obtains the plot on the right.

Which of the following processes gives the biggest contribution to the cross section in this region?



A)



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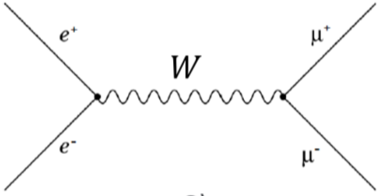

C)

None of them

D)

The plan for today

Momentum conservation,
mass shell

Feynman diagrams

Applying the rules

Fermi’s golden rule

Particles and their interactions

fermions ↓ / bosons →	electro-magnetic	weak		strong
	γ	$W^{\pm}$	Z	g
quarks u c t / d s b				
charged leptons e μ τ				
neutral leptons ν_e ν_μ ν_τ				

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fermions ↓ / bosons →		electro-magnetic	weak		strong
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quarks	**u c t** **d s b**	X	X	X	X
charged leptons	e μ τ	X	X	X	
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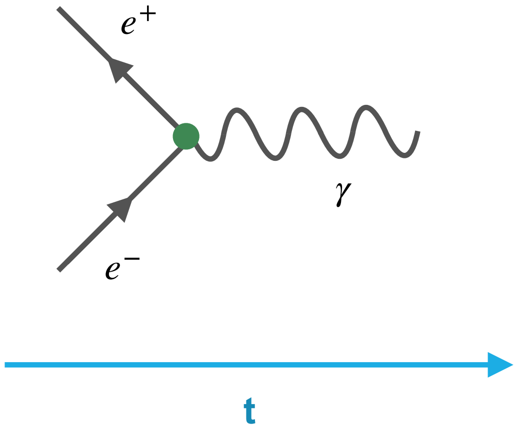
Annihilation of e+ / e- pair

Why can this not be a regular Feynman diagram for the annihilation of a positron and an electron?

A) Due to the arrows' direction charge conservation in the vertex is violated.

B) With the γ as external particle momentum is not conserved in the vertex.

C) Gammas can only exist as virtual particles



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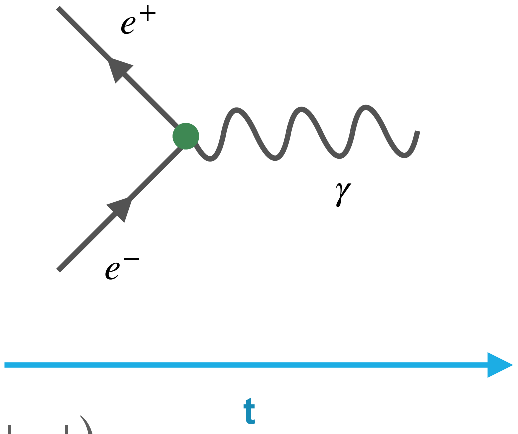

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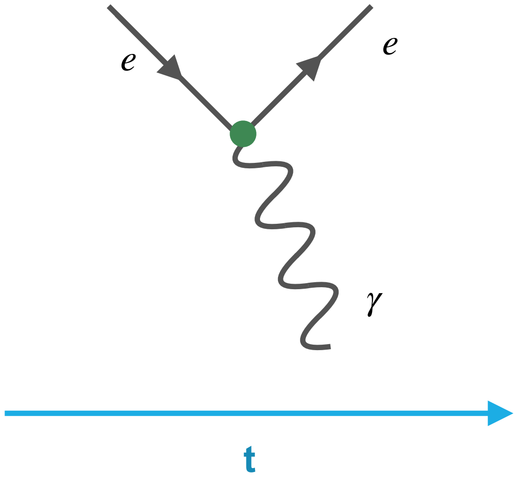
In CMS: $\mathbf{P}_1 + \mathbf{P}_2 = \begin{pmatrix} E_1 + E_2 \\ \vec{0} \end{pmatrix}$ but $\mathbf{P}_\gamma = \begin{pmatrix} |\vec{p}_\gamma| \\ \vec{p}_\gamma \end{pmatrix}$ with $|\vec{p}_\gamma| = E_1 + E_2$

What if we turn the diagram? - Bremsstrahlung?

Is this now a regular diagram?

Argue from the CMS of the incoming electron.

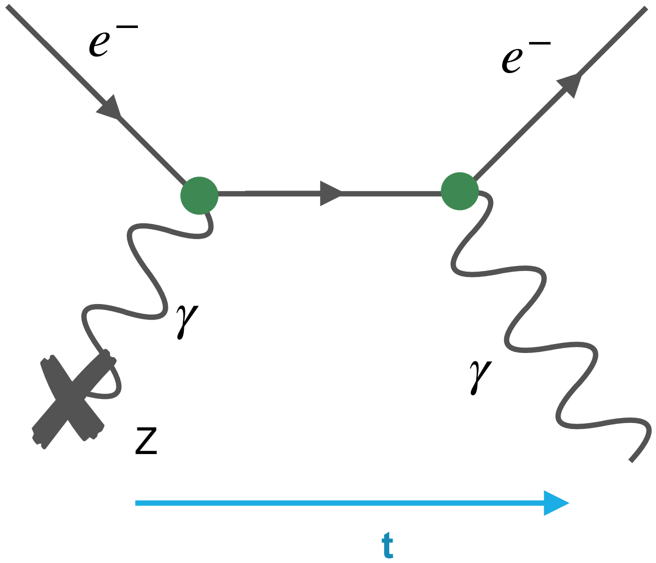


No, not a regular diagram:

In CMS (system of incoming electron) the energy corresponds to the mass m_e.

Therefore it is impossible to produce an additional γ and the momenta for both.

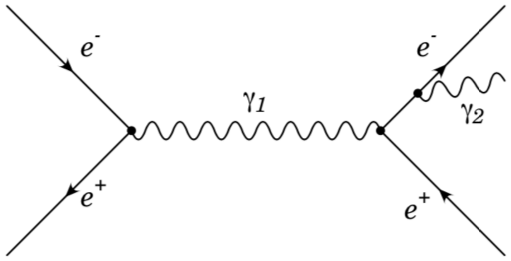
Feynman diagram: Bremsstrahlung



Feynman diagram for bremsstrahlung in field of nucleus.

Interaction with nucleus enables momentum conservation.

Typical exercise: virtual and real photons 1



Which energy, mass and momentum does γ_1 have?
(Expressed in CMS of $e^+ - e^-$ pair)

$$\mathbf{P} = \mathbf{P}_{e^+} + \mathbf{P}_{e^-} = \begin{pmatrix} E_{e^+} + E_{e^-} \\ \vec{p}_{e^+} + \vec{p}_{e^-} \end{pmatrix}$$

$$E_e = \sqrt{m_e^2 + |\vec{p}_e|^2}$$

in CMS: $\vec{p}_{e^+} + \vec{p}_{e^-} = 0$

$$\rightarrow \mathbf{P} = \begin{pmatrix} E_{CMS} \\ \vec{0} \end{pmatrix}$$

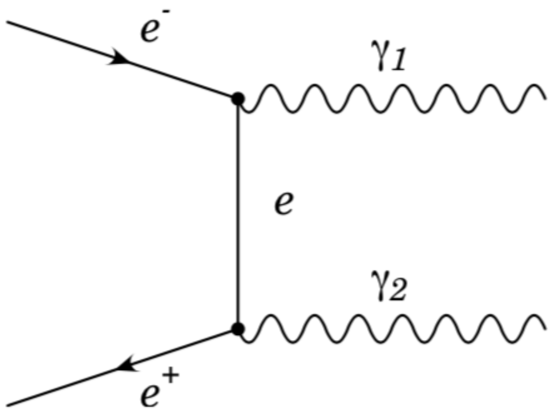
$$\mathbf{P}_{\mathbf{gamma}} = \begin{pmatrix} E_\gamma \\ \vec{p}_\gamma \end{pmatrix} = \begin{pmatrix} \sqrt{m_\gamma^2 + \vec{p}_\gamma^{\,2}} \\ \vec{p}_\gamma \end{pmatrix}$$

Use 4-momentum conservation:

$$\mathbf{P}_{\mathbf{gamma}} = \mathbf{P} \qquad \vec{p}_\gamma = 0 \qquad m_\gamma = E_\gamma = E_{CMS} \neq 0!$$

Meaning: γ_1 has a mass different from 0.
It is “off its mass shell”.

Typical exercise: virtual and real photons 2



Which energy, mass and momentum do these photons have?
(Expressed in CMS of $e^+ - e^-$ pair)

Both γ are real particles, so $m_\gamma = 0$

$$E_{\gamma_1} = E_{\gamma_2} = \left|\vec{p}_\gamma\right| = \frac{E_{e^+} + E_{e^-}}{2}$$

Feynman diagram for annihilation of e^+- e^- pair.
Momentum conservation guaranteed by production of <u>two</u> photons.

Group activity

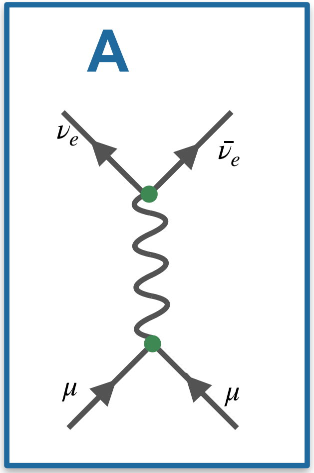
Which statement belongs to which diagram?

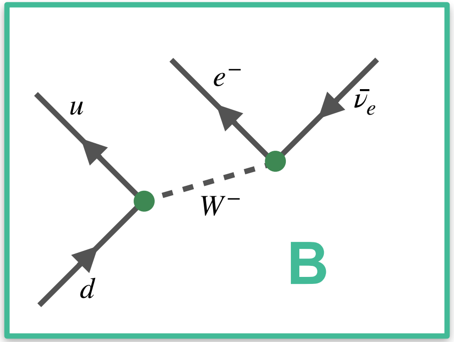
1) β **decay**

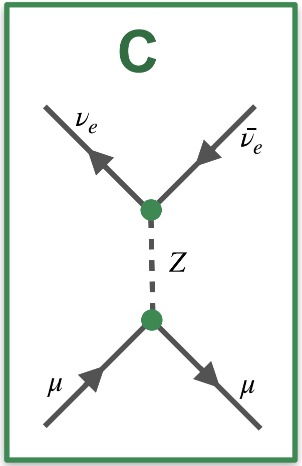
2) **neutrino production**

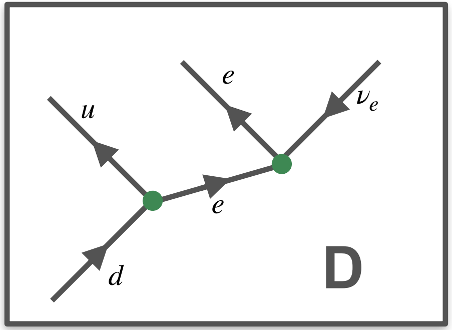
3) **no go:** violates charge conservation and invalid vertex

4) **no go:** invalid vertex









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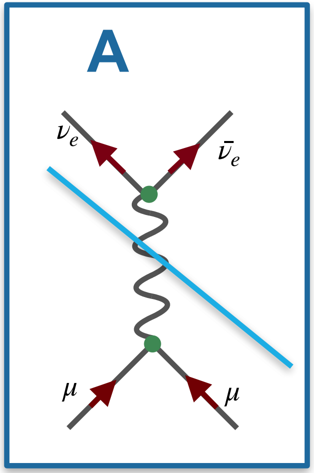
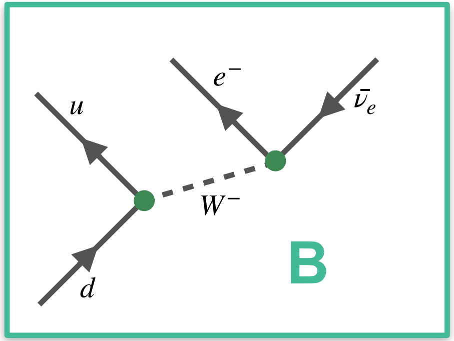
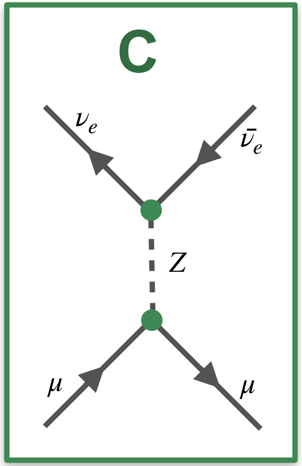
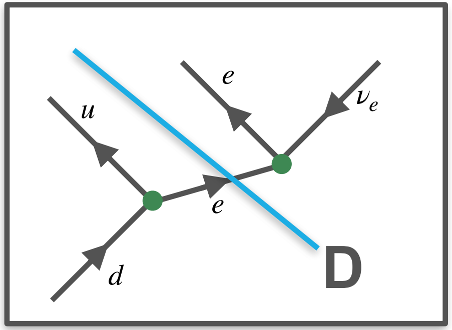




1) β **decay**





4) **no go:** invalid vertex

each vertex needs a boson!

3) **no go:** violates charge conservation
and invalid vertex
(ν only weak interaction!)

2) **neutrino production**

Tipp: Fermion line always goes through

Summary: Interactions in Feynman diagrams

General remarks

- interaction mediated by at least one boson
- most vertices: 2 leptons / quarks, one boson
- each vertex coupling factor $\sqrt{\alpha}$ $\sqrt{\alpha_W}$ $\sqrt{\alpha_s}$

Conservation laws

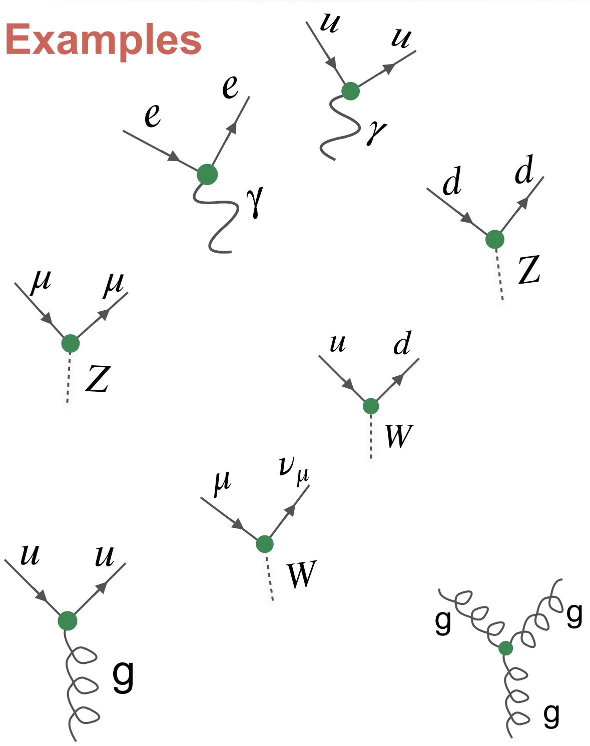
Fundamental conservation laws

electrical charge 4-momentum angular momentum

Empirical findings *exceptions possible!*

lepton number baryon number

Examples



Fermions and (some of) their numbers

		Electric charge	Lepton number	Baryon number
up quarks	u c t	$+\frac{2}{3}e$		
down quarks	d s b			
charged leptons	e μ τ			
neutral leptons	ν_e ν_μ ν_τ		1	0

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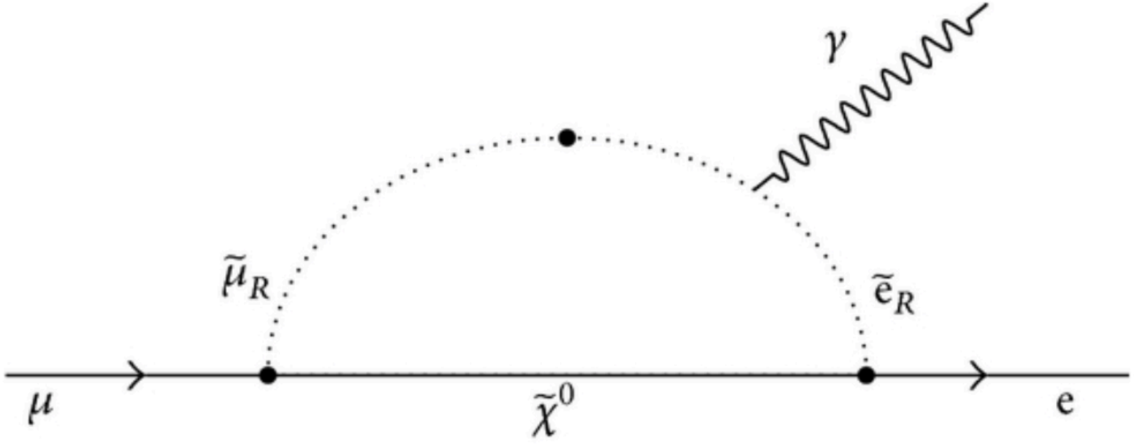
		Electric charge	Lepton number	Baryon number
up quarks	u c t	$+\frac{2}{3}e$	0	$\frac{1}{3}$
down quarks	d s b	$-\frac{1}{3}e$	0	$\frac{1}{3}$
charged leptons	$e \quad \mu \quad \tau$	$-e$	1	0
neutral leptons	$\nu_e \quad \nu_\mu \quad \nu_\tau$	0	1	0

Processes that violate “soft” conservation rules

Imagine someone found new particles that contribute to the depicted process.

Which conservation rule is violated?

How would you name this kind of processes to classify it?



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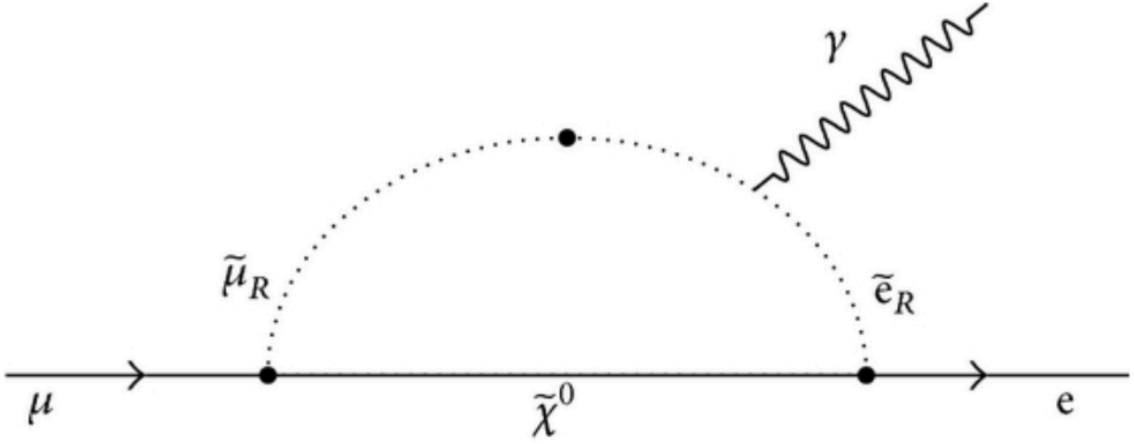
Usual classification:

Charged-lepton-flavour violating process

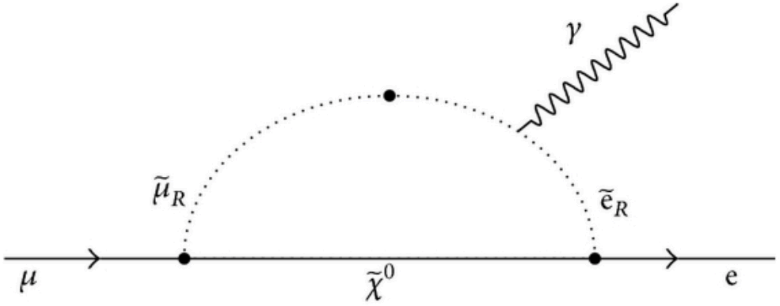


And how can we measure it?

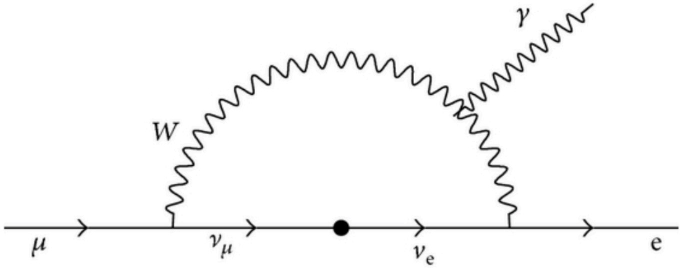
In the Standard Model, it is possible that a muon decays in to $e + \gamma$ (diagram on the right). The SM cross section, however, is extremely low.

Observing muons that decay, how can you get hints of “New Physics”?

$\Rightarrow$ compare rates of $e\gamma$ production with your SM prediction. **“Indirect search”**



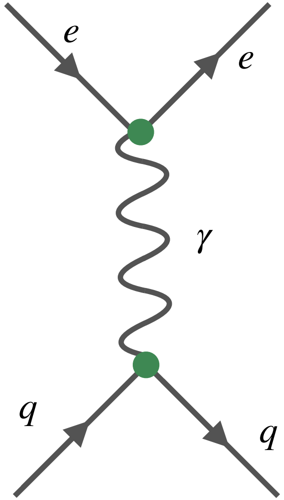
“New Physics”

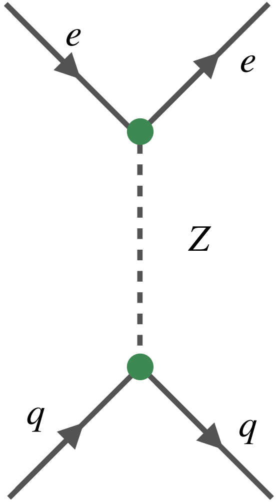


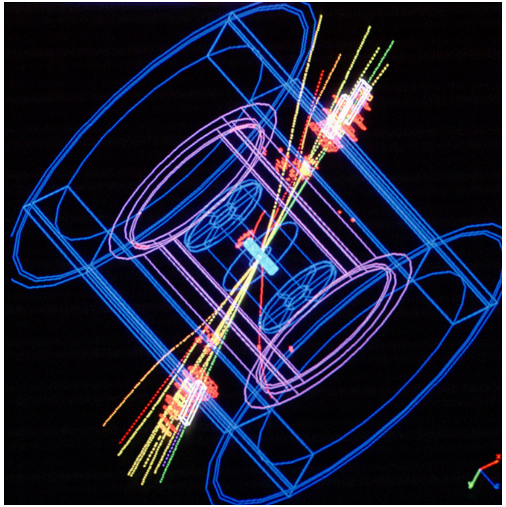
Standard Model (with ν oscillation)

Collider experiment - direct search

At an e+-e- collider we are studying the production of quarks / hadrons.







[CERN-EX-9201024]

Can we find out for a certain event whether it was a Z or a gamma process?

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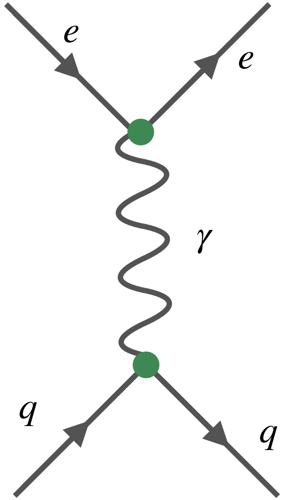
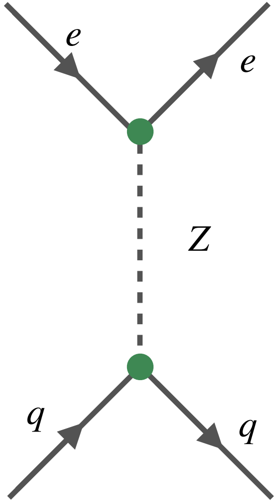
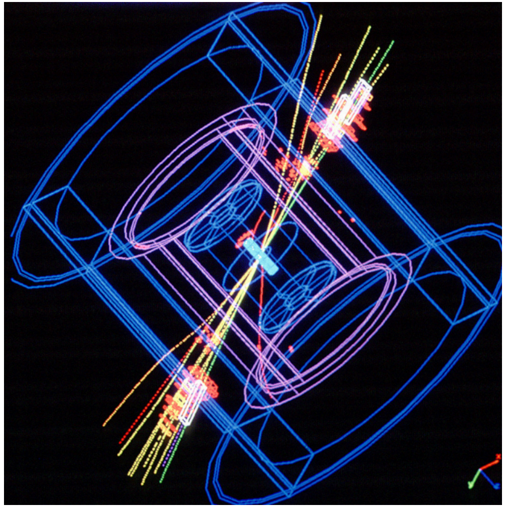






[CERN-EX-9201024]

Can we find out for a certain event whether it was a Z or a gamma process?

Answer:
We can not know it for one single event.
We can only make probabilistic statements.

Fermi's golden rule

$$\Gamma_{i\to f} = \left|M_{fi}\right|^2 \frac{2\pi}{\hbar}\rho_f$$

Transition rate

$e^+ + e^- \rightarrow \text{hadrons}$

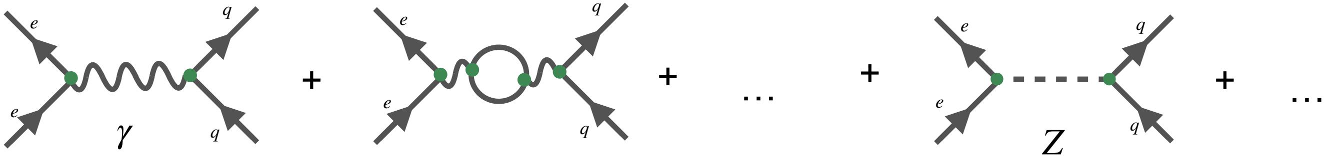
What we measure!

Phase space

How many configurations of momentum in the final state?

Matrix element

contains physics **What we calculate!**



$\Rightarrow$ all Feynman diagrams contribute to our final state.

Adding up Feynman amplitudes

At a collider we study collisions of the type $e^+ + e^- \longrightarrow \mu^+ + \mu^-$.

Explain why or why not the differential cross section can be written as

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{em} + \left(\frac{d\sigma}{d\Omega}\right)_{weak}$$

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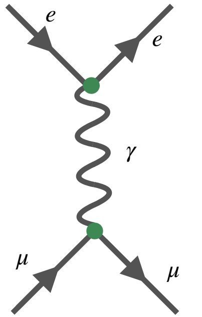
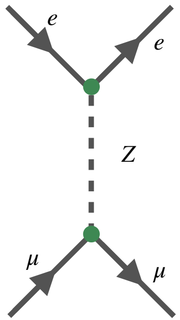




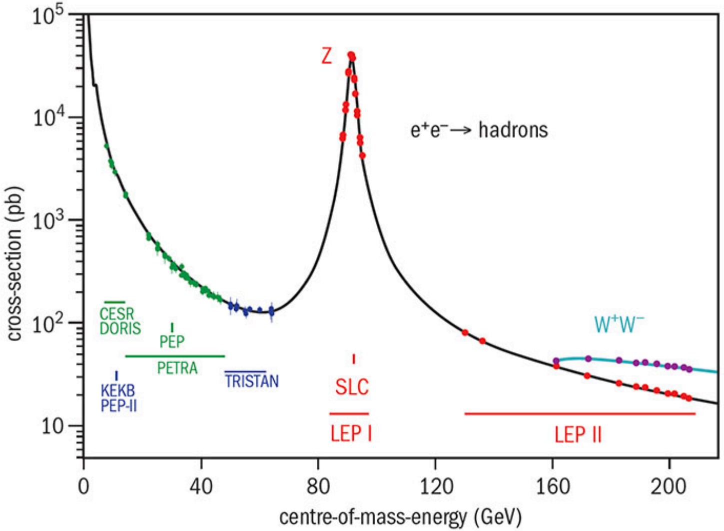
wrong, because: $\frac{d\sigma}{d\Omega} \sim \left|M_{fi}\right|^2$

$M_{em} + M_{weak}$

Diagrams combine already on amplitude level!

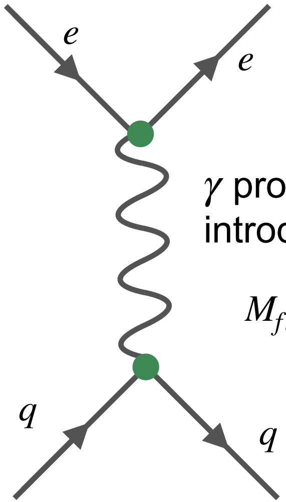
Short outlook: resonances

[arXiv:hep-ex/0509008]



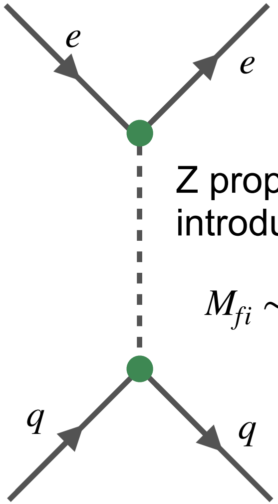
Remember:

Virtual particles can exist off-shell with a mass other than their rest mass.



γ propagator introduces factor

$$M_{fi} \sim \frac{1}{q^2}$$



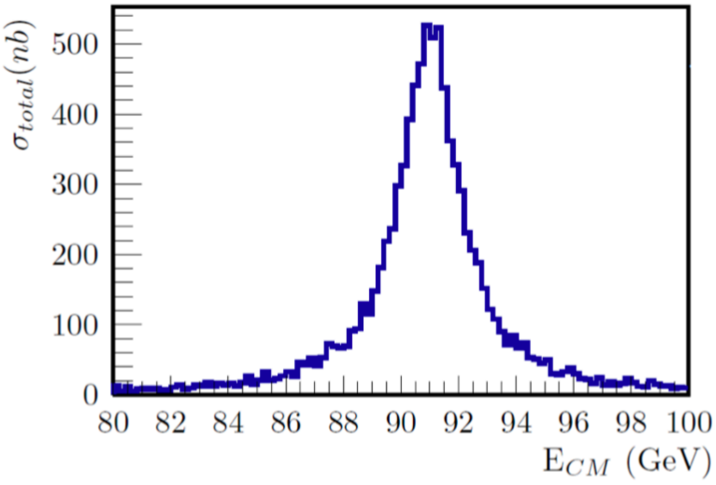
Z propagator introduces factor

$$M_{fi} \sim \frac{1}{q^2 - m^2}$$

But:

the more a virtual particle is off-shell, the less likely is its production

Warmup question



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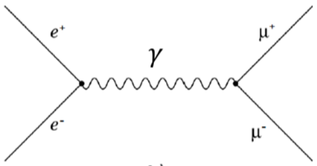

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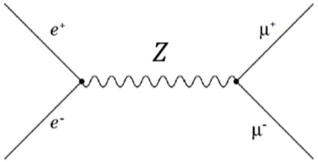

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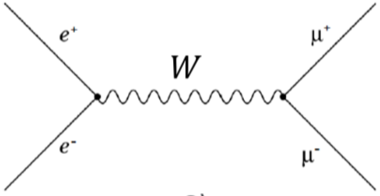

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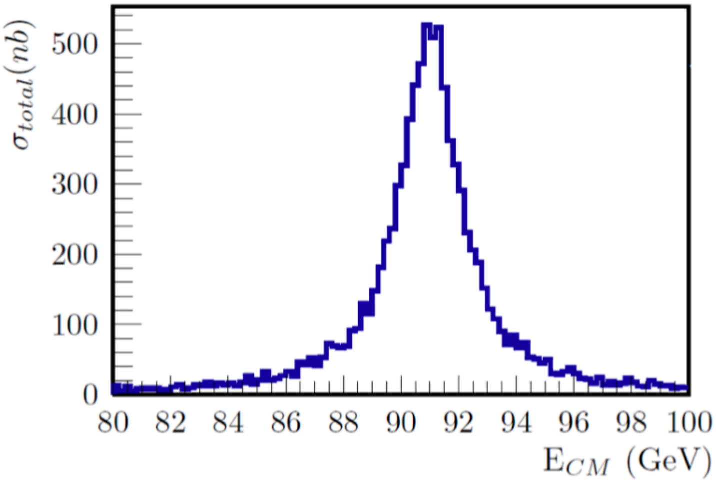

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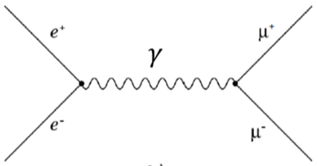

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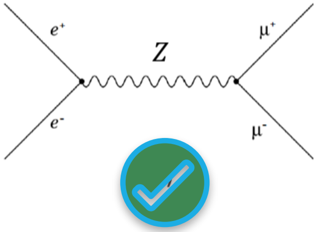
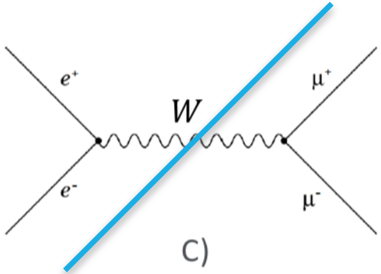


No valid diagram!



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