Introduction to Nuclear and Particle Physics

Lesson 2

reference frames, scattering

PDG booklet

All possible information about particles.

Check it out - it’s good!

https://pdg.lbl.gov

Warm-up question 1

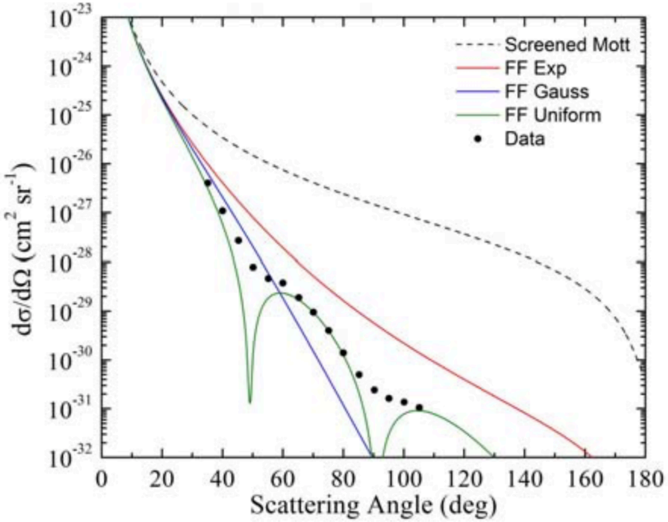
The diagram shows differential cross sections for the scattering of electrons in Indium. It compares measured data to calculations with different assumptions. Which of these statements are correct?

A) The total cross section for the *Screened Mott* calculation is higher than for *FF Uniform*.

B) In the measurement they could have increased $d\sigma/d\Omega$ by measuring longer.

C) The probability for an incoming electron to scatter was measured higher than predicted with the *FF Exp* assumption

Scattering of e$^-$ (183 MeV) in Indium



[A Bagulya *et al* 2017 *J. Phys.: Conf. Ser.* **898** 042032]

Warm-up question 2

CMS = center-of-mass system

Which statements about the center-of-mass are correct?

A) It is impossible to boost into the center of mass of a single photon

B) The invariant mass of two particles is the highest in the CMS.

C) The total momentum $\vec{p}_{CMS} = \vec{p}_1 + \vec{p}_2 + \dots$ in the CMS is always zero.

D) The position of the center of mass is fixed in all frames of reference.

The plan for today

CMS vs LAB

cross sections and scattering measurements

particle scattering

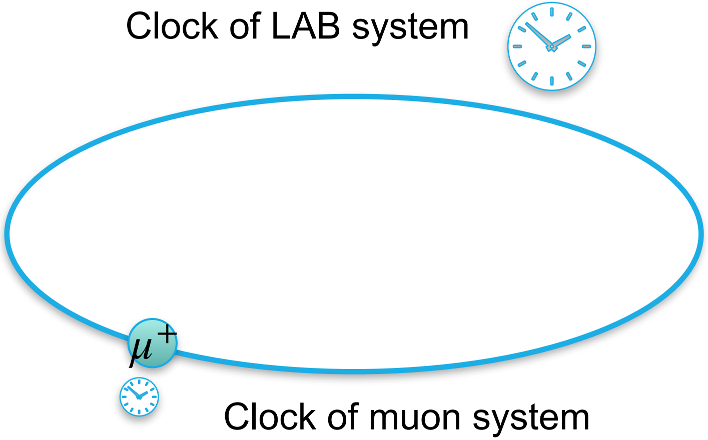
Time dilation

The lifetime of a muon is $\tau = 2.2\ \mu\mathrm{s}$.
Which lifetime τ' would we (LAB frame) measure for a muon cycling in a storage ring at $p = 1\ \mathrm{GeV/c}$?

A) $\tau' < \tau$

B) $\tau' \approx \tau$

C) $\tau' > \tau$



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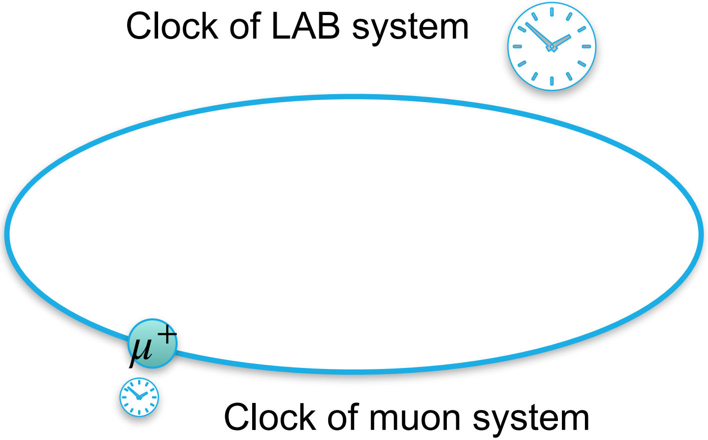


Time dilation! $\tau' = \gamma\tau$
"Time measured in one's rest frame is always shortest."

Muons alive in LAB: $N(t) = N_0\, e^{-\frac{t}{\gamma\tau}}$

Traveled length in LAB: $d = v\tau' = v\gamma\tau$

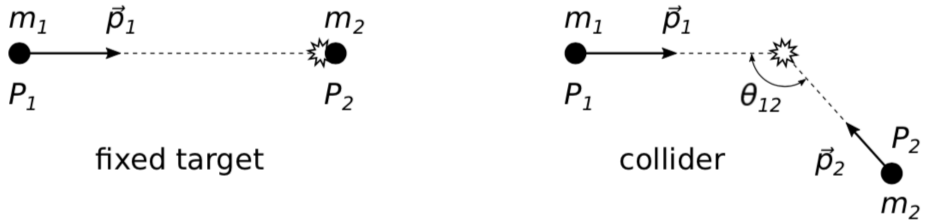
Exercise on center-of-mass energy

fixed target VS collider

$$\sqrt{s} = \sqrt{\mathbf{p}_\mu \cdot \mathbf{p}^\mu}$$

Lorentz invariant

same in all frames!



Procedure to get $\sqrt{s}$:

Starting point for many exercises of this kind!

1) choose frame / be aware of it!

2) determine and sum up 4-momenta

3) square P_{tot} / go on

here: LAB system

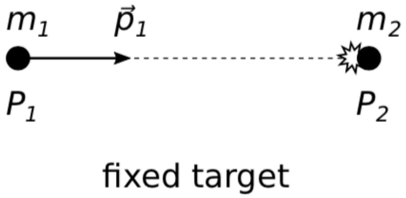
$$P_1 = \left(E_1, \vec{p}_1\right)$$

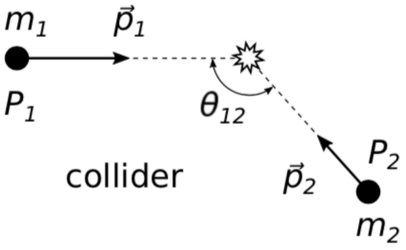
$$P_2 = \left(E_1, \vec{p}_2\right)$$

$$P = \left(E_1 + E_2, \vec{p}_1 + \vec{p}_2\right)$$

So what does it mean?

fixed target VS collider





ultra-relativistic:

$$E \gg m$$

$$\sqrt{s} = \sqrt{2E_1 m_2}$$

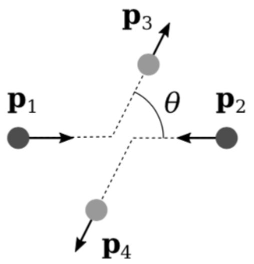
$$\sqrt{s} = \sqrt{4E_1 E_2}$$

[particles head-on ($\theta = 180^o$)]

Fixed target: Lots of energy is needed for momentum of center-of-mass.
$\Rightarrow$ Not available in center-of-mass.

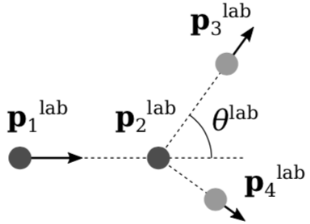
Reference frames: LAB vs CMS

CMS



- Total momentum $\vec{p}_{tot} = ??$

LAB



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Reference frames: LAB vs CMS

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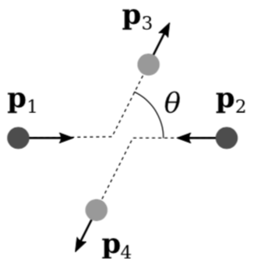
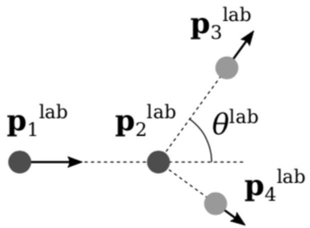


- Total momentum vanishes $\vec{p}_{tot} = 0$
- $2 \rightarrow 2$ collision: always “back to back”
- Single photon: no CMS
  ($E = p,\ m = 0 \Rightarrow$ boost not possible)

LAB



- Any $\vec{p}_{tot}$ possible
  (also 0 for head-on collision)
- **usually:** one particle at rest
  (not in collider experiment)
- Frame of experimentalist

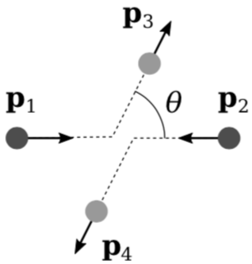
Group activity: CMS $\longleftrightarrow$ LAB   Example for a 2 - 2 collision

Aim: Transfer P_1 between the frames

What are Λ , γ , β ?

CMS

$$\mathbf{P}_1 = \begin{pmatrix} E_1 \\ \vec{p} \end{pmatrix} \qquad \mathbf{P}_2 = \begin{pmatrix} E_2 \\ -\vec{p} \end{pmatrix}$$

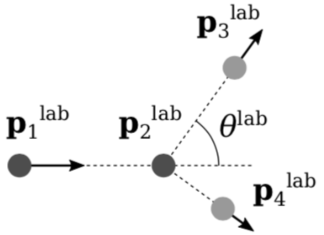


$$\mathbf{P}_1 = \Lambda\ \mathbf{P}_1'$$

LAB

With particle 2 at rest

$$\mathbf{P}_1' = \begin{pmatrix} E_1' \\ \vec{p}\,'_1 \end{pmatrix} \qquad \mathbf{P}_2' = \begin{pmatrix} m_2 \\ \vec{0} \end{pmatrix}$$



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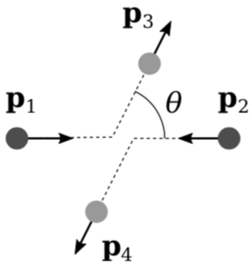

$$\mathbf{P}_1 = \Lambda\ \mathbf{P}_1'$$

$$\mathbf{P}_1' = \Lambda'\ \mathbf{P}_1$$

$$\Lambda = \begin{pmatrix} \gamma & \pm\gamma\beta & 0 & 0 \\ \pm\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\gamma = ??$$

$$\beta = ??$$

LAB

With particle 2 at rest

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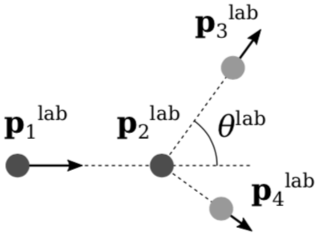

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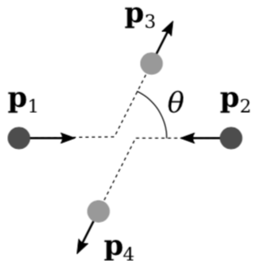

$$\Lambda = \begin{pmatrix} \gamma & +\gamma\beta & 0 & 0 \\ +\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{P}_1 = \Lambda\ \mathbf{P}_1'$$

$$\gamma = \frac{E_2}{m_2} \qquad \beta = \frac{|\vec{p}|}{E_2}$$

$$\mathbf{P}_1' = \Lambda'\ \mathbf{P}_1$$

$$\Lambda' = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

LAB

With particle 2 at rest

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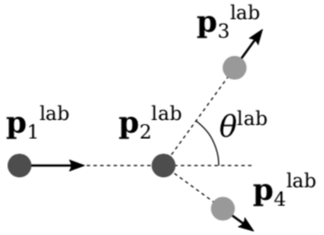
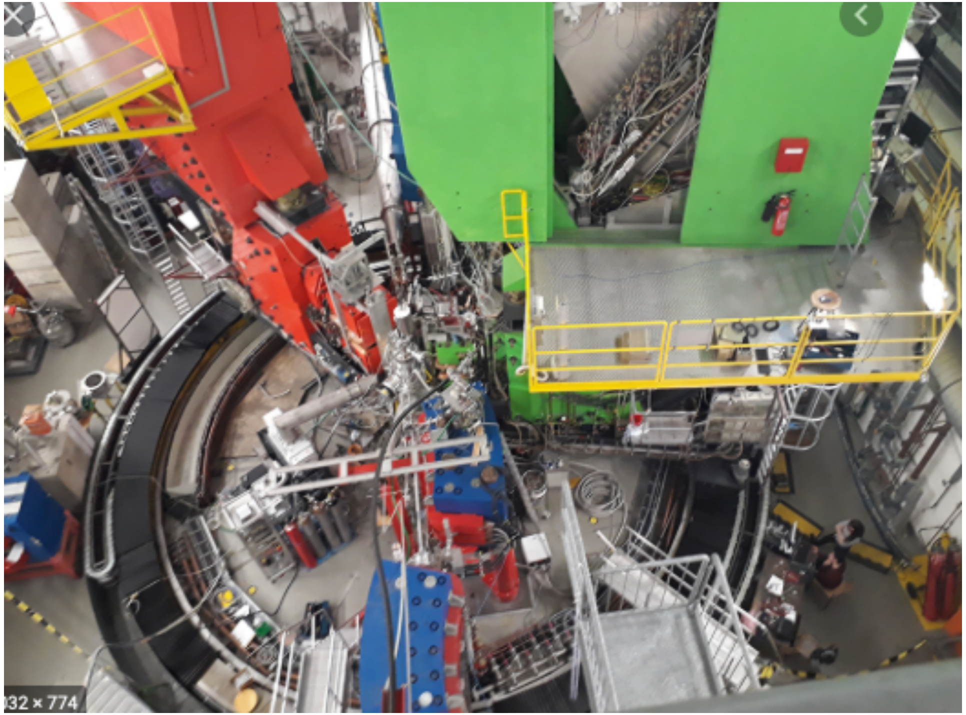
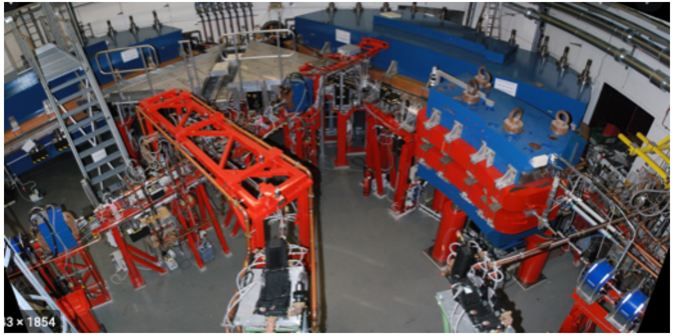
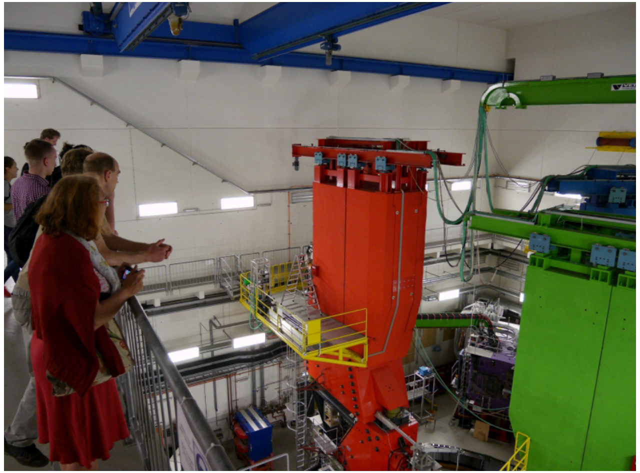


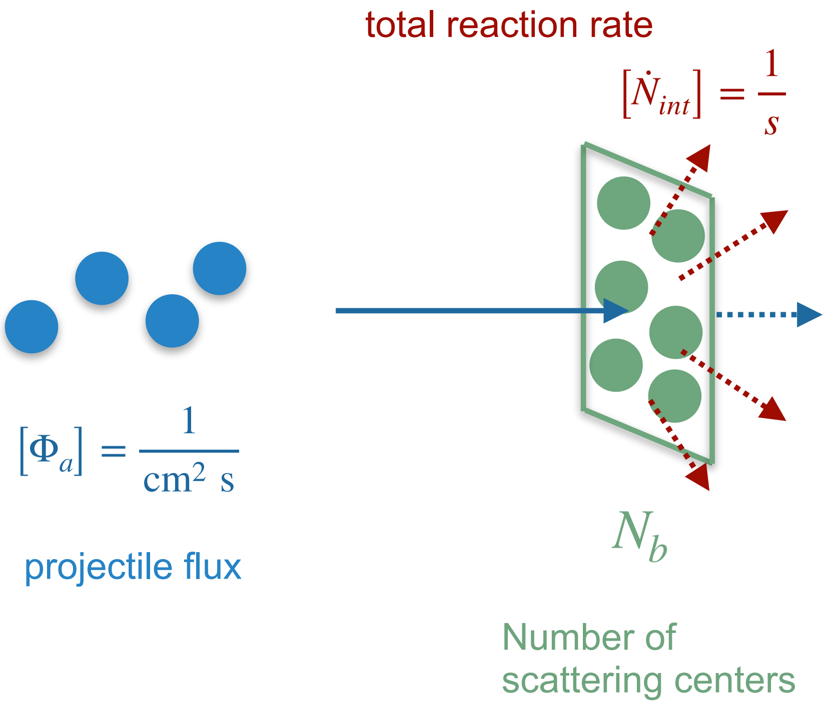
Electron scattering experiment at Mainz Microtron (MAMI)







Scattering experiments and total cross section



What is the probability to scatter the blue projectiles on the green nuclei in the target?

“Naive” geometric approach:

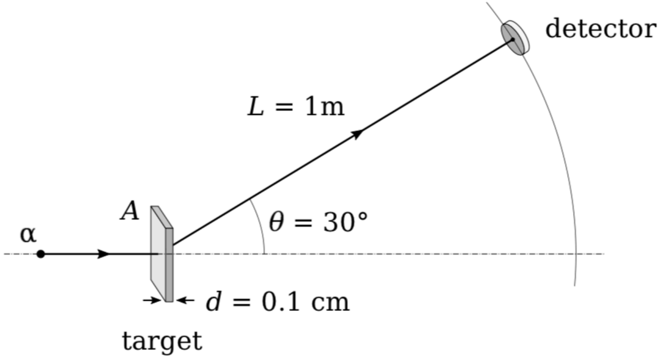
Probability determined by projected area of blue and green particles.

In quantum mechanics:

Cross section is a measure for the probability of a collision

$$\sigma_{AB} = \frac{\dot{N}_{int}}{\Phi_a\ N_b}$$

Differential cross sections



Rate into small solid angle $\Delta\Omega$:

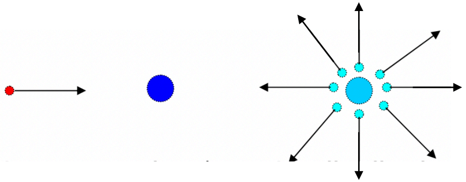
$$\dot{N}(\theta, \Delta\Omega) = \Phi_a N_b \frac{\mathrm{d}\sigma(\theta)}{\mathrm{d}\Omega} \Delta\Omega$$

Luminosity $\mathscr{L} = \Phi_a N_b$

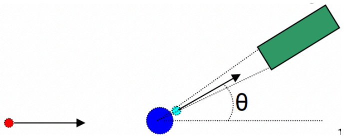
$$\dot{N}_{int} = \mathscr{L}\ \sigma_{tot}$$

How likely is scattering into solid angle $\mathrm{d}\Omega$ at angle θ ?

σ_{tot}



$\dfrac{\mathrm{d}\sigma(\theta)}{\mathrm{d}\Omega}$



$$\sigma_{tot} = \int \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\, d\Omega$$

Warm-up question 1

The diagram shows differential cross sections for the scattering of electrons in Indium. It compares measured data to calculations with different assumptions.
Which of these statements are correct?

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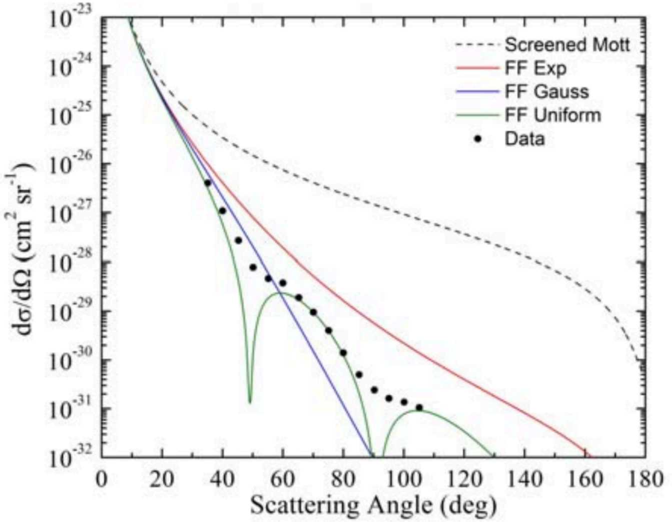
Scattering of e^- (183 MeV) in Indium



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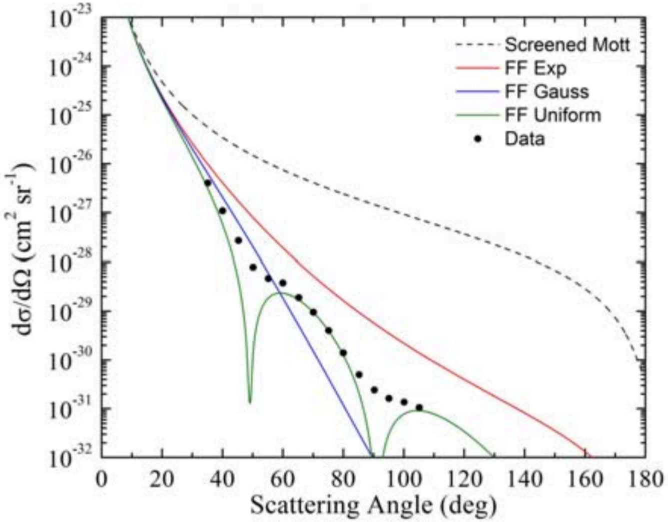

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σ depends only on the scattering partners, not on the geometry or Luminosity!

Warm-up question 2

Which statements about the center of mass are correct?

A) It is impossible to boost into the center of mass of a single photon

B) The invariant mass of two particles is the highest in the CMS.

C) The total momentum $\vec{p}_{CMS} = \vec{p}_1 + \vec{p}_2 + \dots$ in the CMS is always zero.

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$\sqrt{s}$ is Lorentz-invariant!

✓ C) The total momentum $\vec{p}_{CMS} = \vec{p}_1 + \vec{p}_2 + \dots$ in the CMS is always zero.

D) The position of the center of mass is fixed in all frames of reference.

In frames other than the CMS, the position of the center-of-mass is moving