Introduction to Nuclear and Particle Physics

Lesson 12

nuclear shell model

parity of states and particles

Warm-up question 1

Knowing that the nuclear magic numbers are 8, 20, 28, 50 and 82, judge the following statements:

A) The spin-parity of the $^{16}_{8}O$ nucleus is $J^P = 0^+$.

B) The parity of the $^{15}_{7}Ni$ nucleus is even (+1).

C) The neutron absorption cross section of $^{86}_{36}Kr$ is higher than the one of $^{85}_{36}Kr$.

D) The proton separation energy of $^{120}_{50}SN$ is higher than the one of $^{120}_{51}SN$.

E) The proton separation energy of $^{120}_{50}SN$ is higher than the one of $^{122}_{50}SN$.

What do we do today?

Nuclear shell model

Energy states   l-s coupling

Parity & Co

General rules

Quick recap on binding energies

What is the scale of the nuclear binding energy per nucleon?

- ☐ keV.
- ☐ Few MeV.
- ☐ Hundreds of MeV.
- ☐ Few GeV.
- ☐ About 13 eV.
- ☐ Depends on the reference system.

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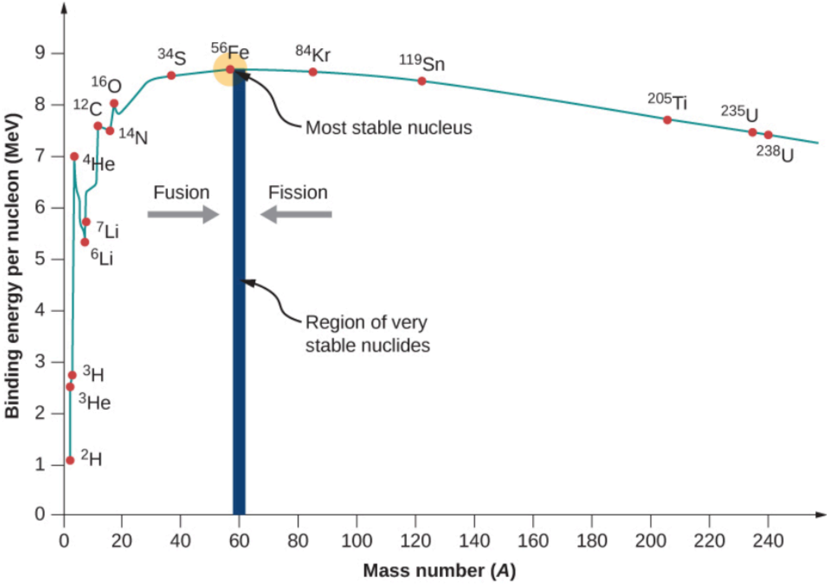
Fission - binding energy



[https://cnx.org/contents/IO_vrGwv@3/Nuclear-Binding-Energy]

What is (roughly) the energy released in a single nuclear fission event of Uranium?

$$E_{1ev} \approx (8-7)\,\frac{\text{MeV}}{\text{Nucleon}} \cdot 200\ \text{Nucleon} = 200\ \text{MeV}$$

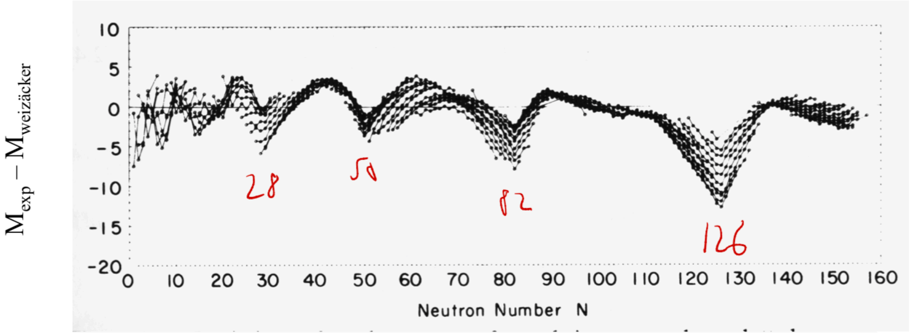
At which rate do fission events take place in a reactor with an output power of $P = 100\ \text{MW}$?

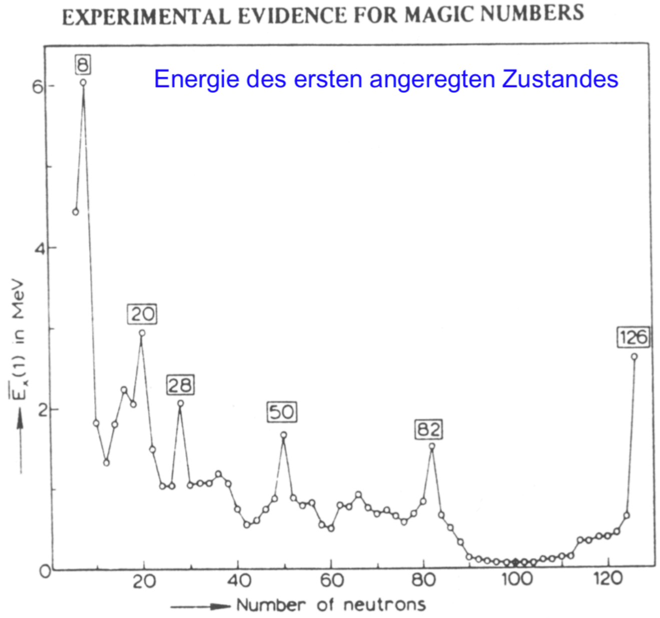
$$200\ \text{MeV} = 2\cdot 10^{8}\cdot 1.6\cdot 10^{-19}\ \text{J} = 3.2\cdot 10^{-11}\ \text{J}$$

$$\text{rate} = \frac{P}{E_{1ev}} = \frac{10^{8}\ \text{W}}{3.2\cdot 10^{-11}\ \text{J}} = 3\cdot 10^{18}\ \frac{1}{\text{s}}$$

Shell model and energy states

Motivation for the nuclear shell model





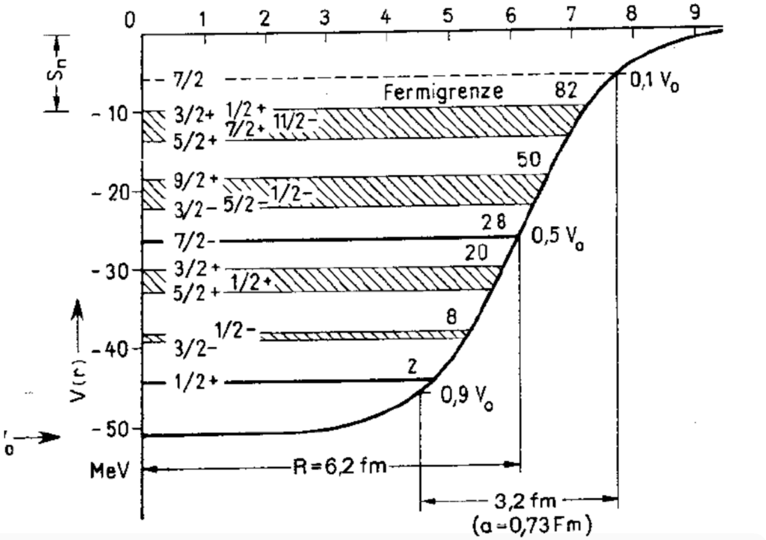
Observations:

Nuclei with specific numbers of p⁺ / n come with:

- specifically high binding energies
- large energy gaps to first excitation

*“**Magic numbers”** point to closed shells of nucleons!*
(analogously to atomic shells)

The Woods-Saxon potential



$$V(r) = -\frac{V_0}{1+e^{\frac{r-R}{a}}}$$

mean-field potential:

- Potential which nucleon sees is created by other nucleons
- Removal of nucleons / decay changes the potential!

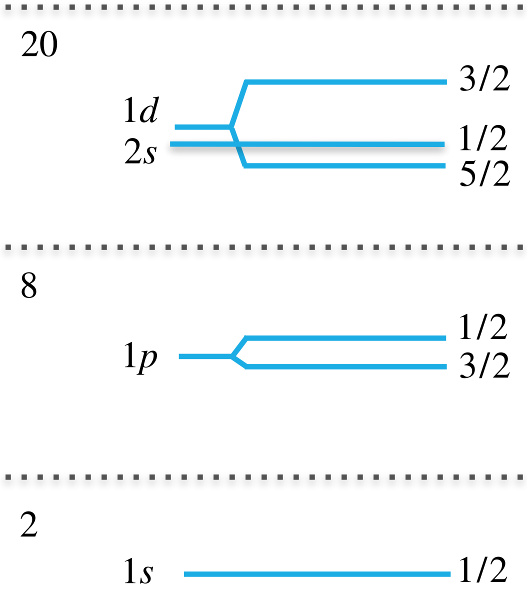
Solution of stationary Schrödinger equation for spherically symmetric potential:

$$E\psi = H\psi$$

Factorization in radial and angular part

$$R_{nl}(r) \qquad Y_l^m(\theta,\phi)$$

Spin-orbit coupling and nucleon energy levels



Lower single nucleon energy states
(p⁺ and n differ slightly at higher energies)

l-s coupling

possible values of quantum numbers:

$$n = 1 \ldots \infty \qquad s = \frac{1}{2} \qquad j = l \pm \frac{1}{2}$$
$$l = 0 \ldots \infty$$

Fine structure turns out to be much stronger than in atoms!

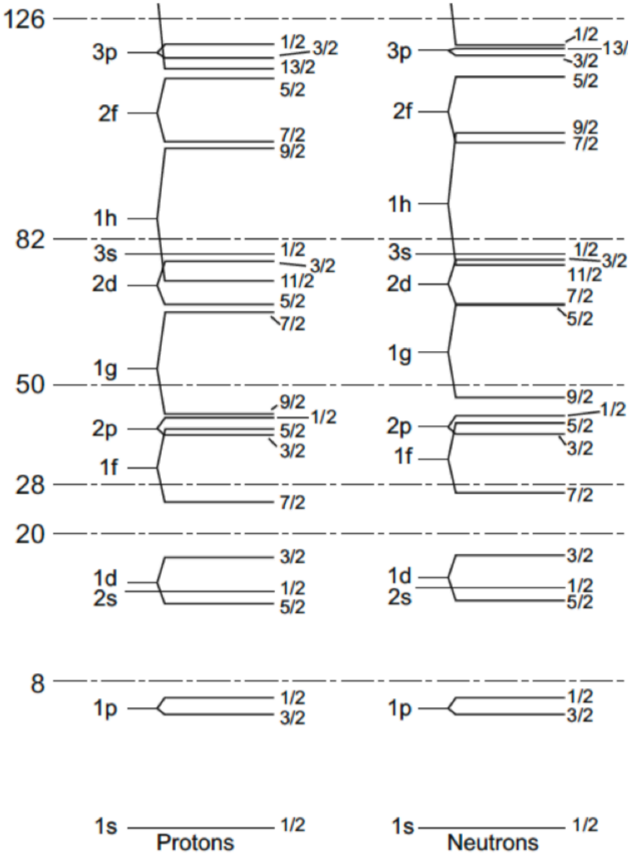
Remarks on parity:

- Each nucleon state brings in $(-1)^l$ to the total parity.
- Full shells have J=0, parity +1 and $\mu = 0$.
- Unpaired nucleon states define J and parity of whole nucleus

More energy levels…

Examples:

- Ground state of ${}^{17}_{8}\mathrm{O}_9$ (8 protons, 9 neutrons) is defined by single neutron in 1d$^{5/2}$: $J^P = 5/2^+$
- Ground state of ${}^{3}_{2}\mathrm{He}_1$ (2 protons, 1 neutron) is defined by missing neutron in 1s$^{1/2}$: $J^P = 1/2^+$

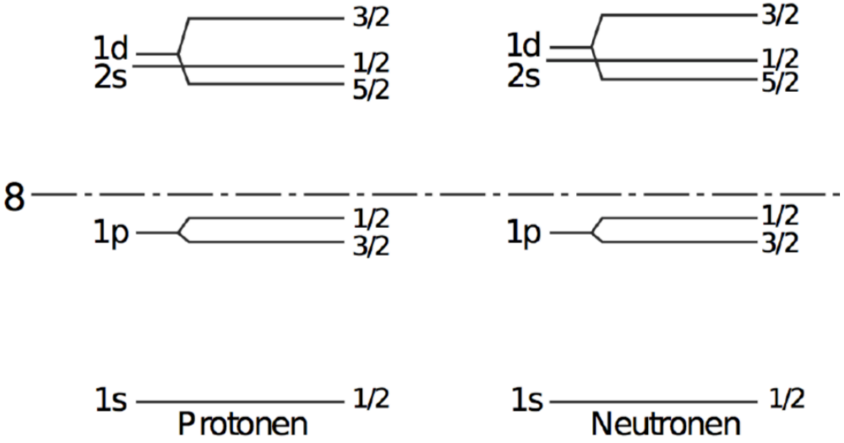


Group activity

Please answer the following questions together in groups.
Be ready to discuss your results in the plenary later-on.

Question 1

What is the maximal number of neutrons that can be in the 1d energy levels of a nucleus?

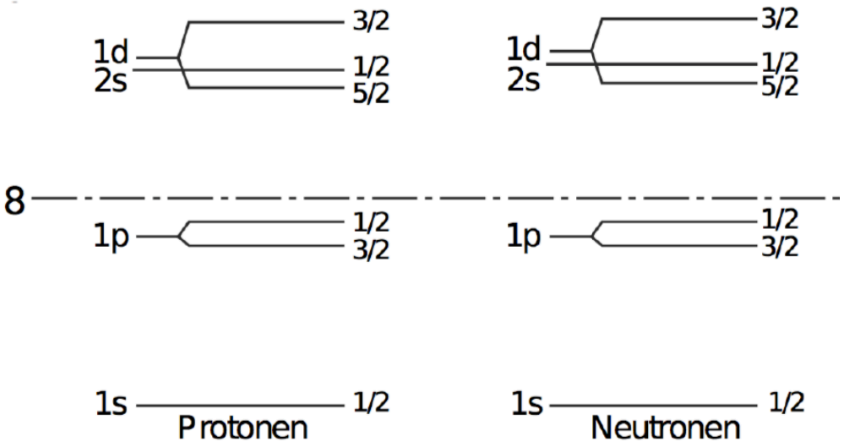


Question 1

What is the maximal number of neutrons that can be in the 1d energy levels of a nucleus?

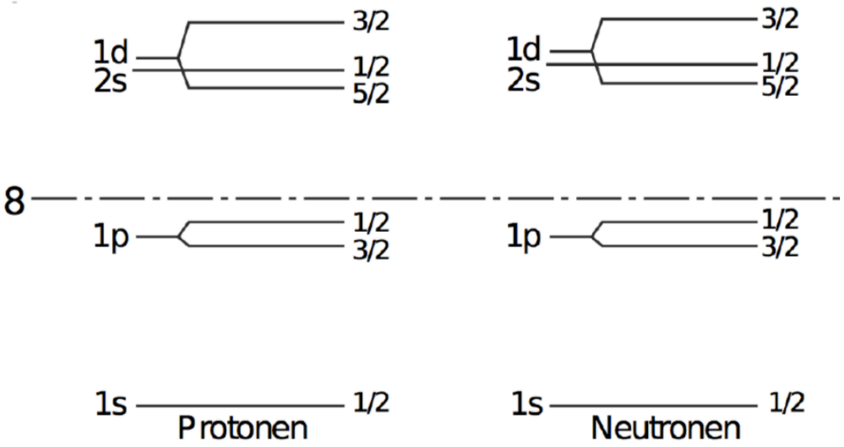
The number if neutrons per j state is 2j+1.

Therefore, 6+4=10 neutrons can be in the two 1d levels.



Question 2

Give the spin, parity and magnetic moment of the ${}^{16}_{8}O_8$ nucleus (Notation ${}^{A}_{Z}O_N$).



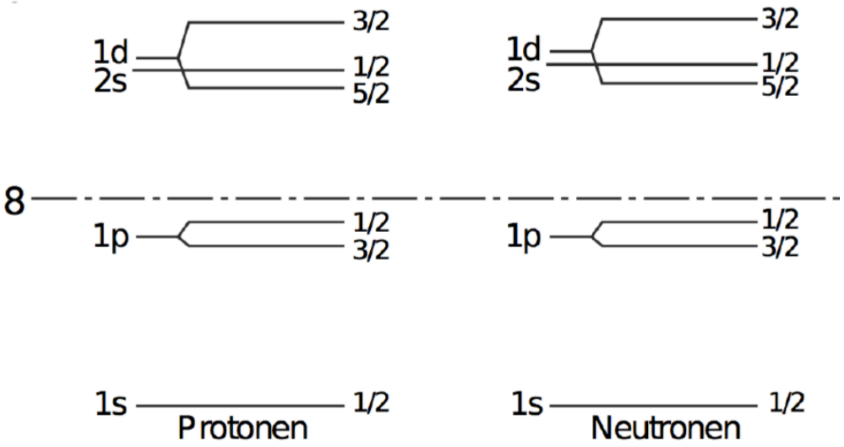
Question 2

Give the spin, parity and magnetic moment of the ${}^{16}_{8}\mathrm{O}_8$ nucleus (Notation ${}^{A}_{Z}\mathrm{O}_N$).

Closed shell !

$$J^P = 0^+$$

$$\mu = 0$$

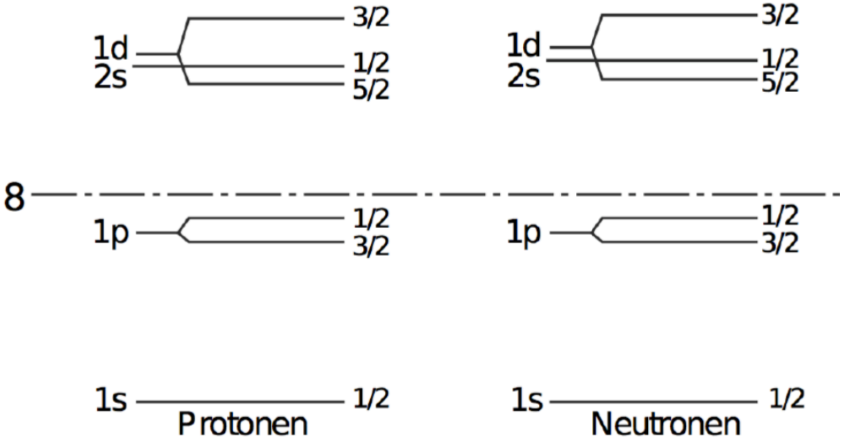


Parity can also be calculated level-wise via

$$P\,\left|n,j,l\right\rangle = (-1)^l \left|n,j,l\right\rangle$$

Question 3

Give the spin, parity and magnetic moment of the ${}^{17}_{8}O_9$ nucleus.



Question 3

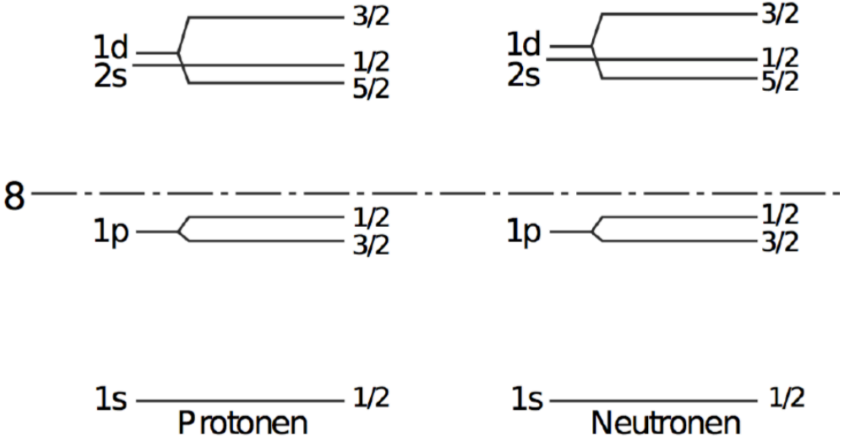
Give the spin, parity and magnetic moment of the $^{17}_{8}O_9$ nucleus.

One neutron in $1d^{5/2}$ remains on top of closed shells.

$$J^P = \frac{5}{2}^+ \qquad l = 2$$

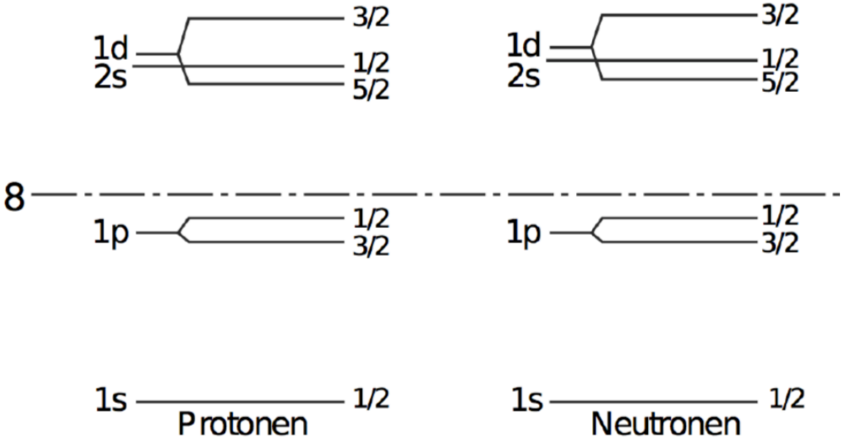
Since the neutron is uncharged, the magnetic moment is given by its own magnetic moment (no contribution to μ from orbital momentum)

$$\mu = \mu_{\text{Neutron}} = -1.91\ \mu_N \qquad \mu_N = \frac{e\hbar}{2m_p}$$



Question 4

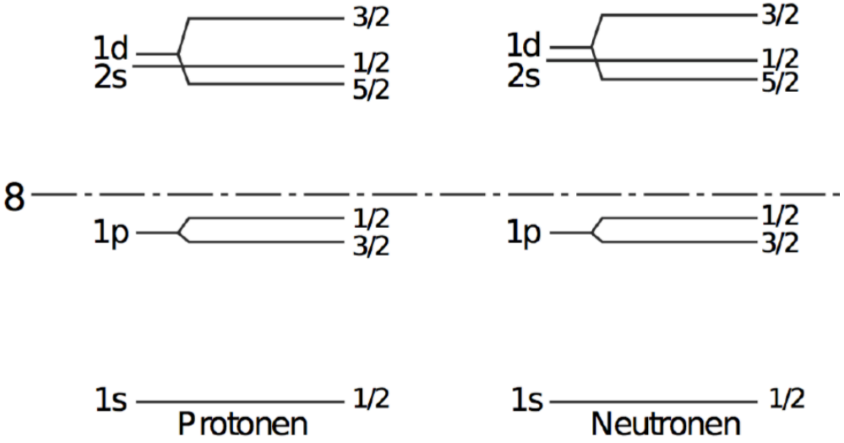
Give the spin and parity of the first excited state of the ${}^{17}_{8}\mathrm{O}_9$ nucleus.



Question 4

Give the spin and parity of the first excited state of the $^{17}_{8}O_9$ nucleus.

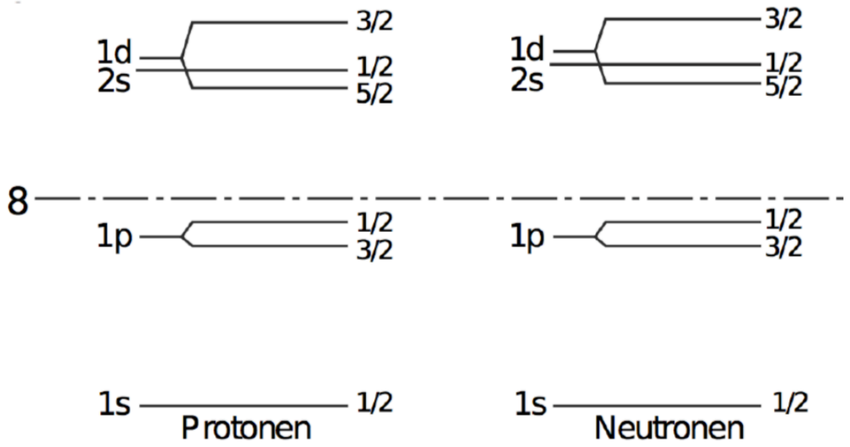
$l = 0 \qquad j = 1/2 \qquad \rightarrow \qquad J^P = 1/2^+$



The first excitation is given by the neutron going from $1d^{5/2}$ to $2s^{1/2}$

Question 5

Give the spin and parity of the ${}^{15}_{7}N_8$ nucleus.

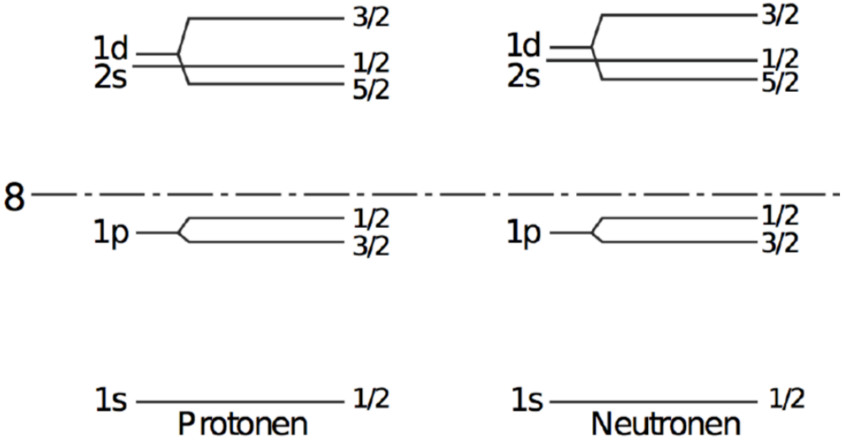


Question 5

Give the spin and parity of the ${}^{15}_{7}N_8$ nucleus.

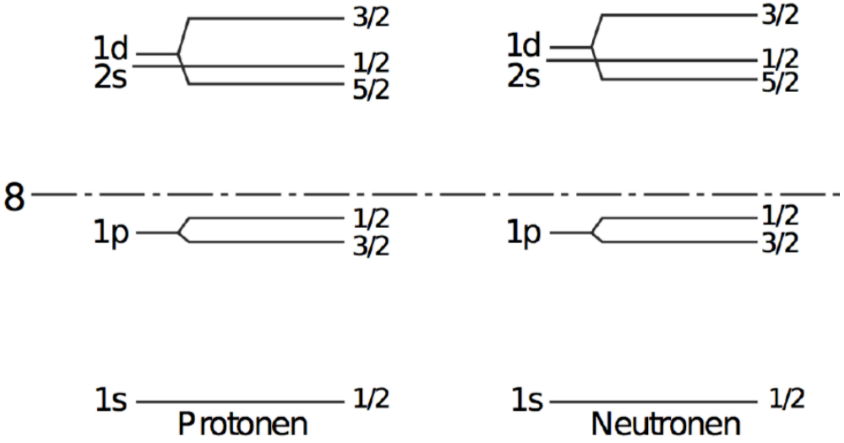
One neutron in $1p^{1/2}$ remains un-paired!

$l = 1$ $j = 1/2$ → $J^P = 1/2^-$



Question 6

Give the spin and parity of the first excited state of the ${}^{15}_{7}\mathrm{N}_8$ nucleus.



Question 6

Give the spin and parity of the first excited state of the ${}^{15}_{7}\mathrm{N}_8$ nucleus.

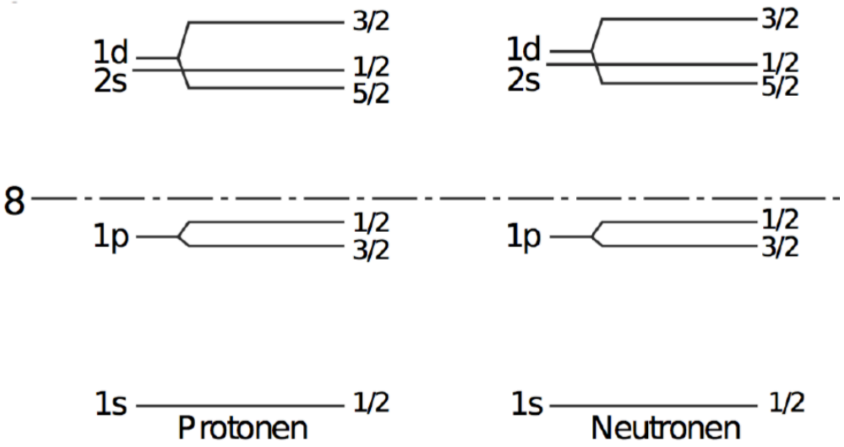
First excitation should be $1p^{3/2} \rightarrow 1p^{1/2}$

Then: un-paired neutron in $1p^{3/2}$.

$l = 1 \qquad j = 3/2 \qquad \rightarrow \qquad J^P = 3/2^-$

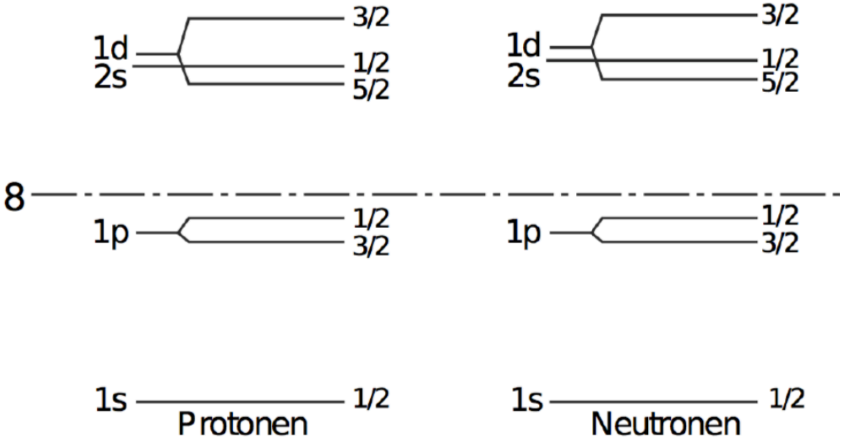
But: Measurement gives first three excitations as

$$J^P = 5/2^+,\ 1/2^+,\ 3/2^-$$



Question 7

Why is the binding energy of ${}^{14}_{8}\mathrm{O}_6$ smaller than that of ${}^{14}_{7}\mathrm{N}_7$?



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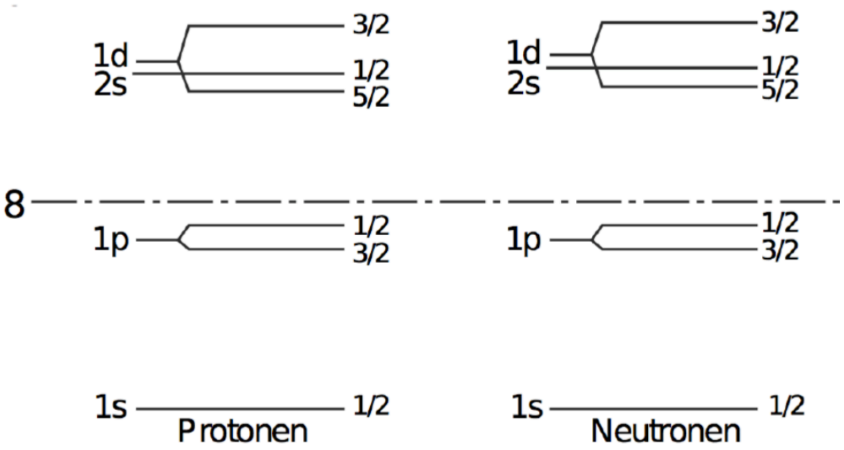
Coulomb repulsion of more protons leads to lower binding energies.



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B) The parity of the ${}^{15}_{7}Ni$ nucleus is even (+1).

is odd (unpaired proton in $1p^{1/2}$ orbital)

C) The neutron absorption cross section of ${}^{86}_{36}Kr$ is higher than the one of ${}^{85}_{36}Kr$.

Is lower! (nucleus is happy with 50 neutrons)

✓ The proton separation energy of ${}^{120}_{50}SN$ is higher than the one of ${}^{120}_{51}SN$.

E) The proton separation energy of ${}^{120}_{50}SN$ is higher than the one of ${}^{122}_{50}SN$.

Is lower! (more neutrons -> more glue)