

# Introduction to Nuclear and Particle Physics

## Lesson 11

*Standard Model*

*Calculations with CP and P*

*Introduction to nuclear physics*

# Warm-up question 1

Which statements about the standard model are correct?

- A) The Higgs boson can interact with all particles of the SM.
- B) The CP symmetry is slightly violated in the weak interaction.
- C) Parity violation can be observed, but is not part of the Standard Model.
- D) The top quark couples stronger to the Higgs than all other quarks.
- E) The fermion masses are a direct consequence of the symmetry breaking in the SM.

## Warm-up question 2

Which statements about atomic nuclei are correct?

- A) Large, stable nuclei contain more neutrons than protons.
- B) Nuclear fission events free more energy than nuclear fusion events.
- C) The nucleons in the deuteron ( ${}^2\text{H}$ ) are stronger bound than the nucleons in the  ${}^{238}\text{U}$  nucleus
- D) The small mass of the neutrino strongly influences the energy spectrum of  $\beta$  radiation.

# What do we do today?

## The Standard Model

Symmetries

Sectors and couplings of the SM

## Nuclear physics

Introduction on Binding energy  
+ nuclear models

## Calculations with P and CP

# Standard Model activity

# Some statements about the Standard model

1

2

3

4

5

6

7

8

9

10

11

12

13

# Some statements about the Standard model

Differently to the W boson, the Z boson can couple also to right-chiral fermions.

The W boson can mix different quark generations

The Higgs boson gives masses only to fermions

The neutrino masses are zero in the SM

SM interactions do not conserve charge

Helicity is Lorentz-invariant.

Gravitation is not unified in the SM

The Higgs boson couples to gluons

The CP symmetry is broken in the electro-weak sector

The coupling strength between Higgs and a fermion is determined by the fermions mass.

The Higgs boson couples to all particles of the SM

Interactions are carried by 12 bosons in the SM

All coupling constants are equal in the SM.

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# The sectors of the Standard Model

## Standard Model

### QCD sector

color

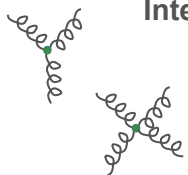
$g_s$

*strong*

**gauge bosons:** 8 gluons

**Interactions:**

gluon - gluon  
quark - gluon  
(and more)



### Electroweak sector

*weak + electromagnetic*

Hypercharge

Weak isospin

$g_w$

$Q_e$

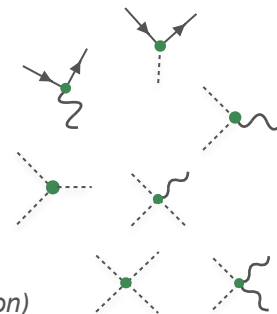
**gauge bosons:**  $W^\pm, Z, \gamma$

**Interactions:**

bosons - fermions  
bosons - bosons (and more)

*Special for weak component:*

- Quark mixing (CKM)
- Chiral theory (Parity violation)



### Yukawa sector

$m_f$

**mass terms** for fermions (not for  $\nu$ )

**Interactions:**

Higgs - fermions



### Higgs sector "Symmetry breaking"

$m_Z$

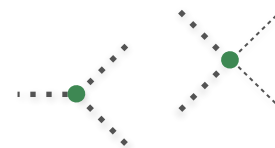
$m_W$

$m_h$

**mass terms** for Higgs, W, Z

**Interactions:**

Higgs - W Higgs - Higgs  
Higgs - Z (and more)



# Group work on parity

**Please solve the exercise of your group together.  
You can get help from the solution presented below your exercise.**

**Be ready to present and explain the solution to the others afterwards.**

## Group work on parity - Exercise 1

The parity of the pion was measured in 1954 by stopping negative pions in a deuteron target, they form a pionic atom and the pion decays into the atomic S-state before the reaction  $\pi^- d \rightarrow nn$  takes place.

- a) The deuteron  $d$  spin is 1, the neutron spin is  $\frac{1}{2}$  and the  $\pi$  spin was known to be 0. What is the total angular momentum  $J$  of the initial state?
  
- b) What are the possible  $L - S$  combinations for the final state?

# Group work on parity - Exercise 1

The parity of the pion was measured in 1954 by stopping negative pions in a deuteron target, they form a pionic atom and the pion decays into the atomic S-state before the reaction  $\pi^- d \rightarrow nn$  takes place.

- c) The final state must be antisymmetric under the exchange of the 2 neutrons. The overall symmetry factor associated to the spin and orbital components is

$$\eta_{LS} = (-1)^{L+S+1} \quad (1)$$

Which among the  $L - S$  combinations for the final state satisfy the antisymmetry requirement?

- d) The parity of a 2-particle state is  $(-1)^L$  times the intrinsic parities.  $d$  and  $n$  have positive parities: what is the parity of the  $\pi$ ?

## Group work on parity - Exercise 1 Solution

The parity of the pion was measured in 1954 by stopping negative pions in a deuteron target, they form a pionic atom and the pion decays into the atomic S-state before the reaction  $\pi^- d \rightarrow nn$  takes place.

- a) The deuteron  $d$  spin is 1, the neutron spin is  $\frac{1}{2}$  and the  $\pi$  spin was known to be 0. What is the total angular momentum  $J$  of the initial state?

$$J = 1.$$

- b) What are the possible  $L - S$  combinations for the final state?

$$J = 1 \rightarrow S = 0, L' = 1$$

$$S = 1, L' = 0, 1, 2$$

# Group work on parity - Exercise 1 Solution

The parity of the pion was measured in 1954 by stopping negative pions in a deuteron target, they form a pionic atom and the pion decays into the atomic S-state before the reaction  $\pi^- d \rightarrow nn$  takes place.

- c) The final state must be antisymmetric under the exchange of the 2 neutrons. The overall symmetry factor associated to the spin and orbital components is

$$\eta_{LS} = (-1)^{L+S+1} \quad (1)$$

Which among the  $L - S$  combinations for the final state satisfy the antisymmetry requirement?

- c) Imposing  $\eta_{LS} = -1$  it follows that the only allowed combination is  $S = 1, L' = 1$

# Group work on parity - Exercise 1 Solution

The parity of the pion was measured in 1954 by stopping negative pions in a deuteron target, they form a pionic atom and the pion decays into the atomic S-state before the reaction  $\pi^- d \rightarrow nn$  takes place.

- d) The parity of a 2-particle state is  $(-1)^L$  times the intrinsic parities.  $d$  and  $n$  have positive parities: what is the parity of the  $\pi$ ?
- d) The initial and the final state have the same parity, which is conserved by the strong interaction. The parity of the final state, for  $S = 1$ ,  $L' = 1$ , is  $\eta_n \eta_n (-1)^{L'} = -1$ . In the initial state  $\eta_\pi \eta_d (-1)^L = \eta_\pi = -1$ . Thus, the  $\pi$  has negative parity.

## Group work on parity - Exercise 2

Typically, neutral kaons decay weakly in 2 pions or in 3 pions. Assuming the CP conservation in the weak sector, find out which are the allowed pion decay modes of  $K_1$  and  $K_2$ . Proceed in the following steps:

Consider the possible 2 pion decay mode with final state  $\pi_0\pi_0$  and determine the corresponding CP eigenvalue.

Hint: You can assume  $P \left| \pi_0 \right\rangle = -1 \left| \pi_0 \right\rangle$



## Group work on parity - Exercise 2 Solution

b) The  $\pi^0$  is a spin zero meson. It is an eigenstate of C, P and CP transformations

$$C|\pi^0\rangle = |\pi^0\rangle \quad (\text{proof it using the quark compositions of the pion}) \quad (11)$$

$$P|\pi^0\rangle = -1|\pi^0\rangle \quad (\text{see Series 9, Exercise 1}) \quad (12)$$

so that

$$CP|\pi^0\rangle = -1|\pi^0\rangle \quad (13)$$

Hence, the pion CP eigenvalue is:

$$CP(\pi^0) = C(\pi^0)P(\pi^0) = (+1) \cdot (-1) = -1 \quad (14)$$

## Group work on parity - Exercise 2 Solution

The system of 2  $\pi^0$ s is a bosonic system, symmetric under particle exchange, with total electrical charge and magnetic moment 0. Therefore, it is also a C, P and CP eigenstate with CP parity:

$$CP(\pi^0\pi^0) = C(\pi^0\pi^0) \cdot P(\pi^0\pi^0) = (+1)(+1) \cdot (-1)(-1)(-1)^L = +1 \quad (15)$$

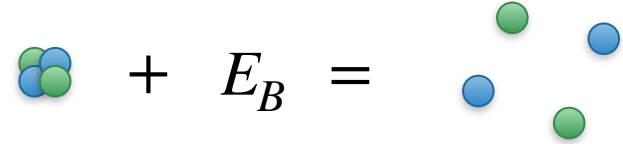
where we used that  $P(\pi^0\pi^0) = P(\pi^0)P(\pi^0)(-1)^L$  and  $L = 0$  from conservation of the total angular momentum from the Kaon decay (kaon spin is zero, pion spin is zero):

$$K^0 \rightarrow \pi^0 + \pi^0 \quad (16)$$

$$0 \rightarrow 0 + 0 + L \quad (\text{conservation of angular momentum}) \quad (17)$$

# Nuclear physics

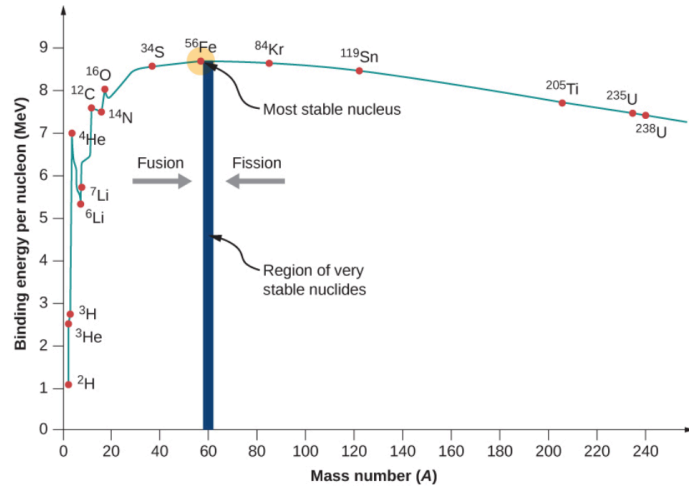
# Binding energy of nuclei



Observation:

The mass of atomic nuclei is smaller than the mass of the contained nucleons.

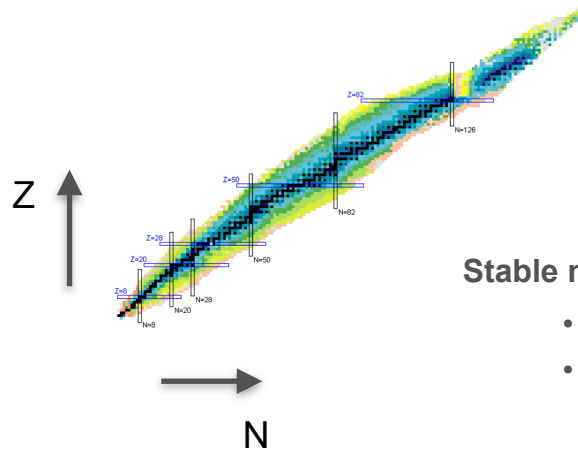
*Binding energy per nucleon describes how deep nucleons are in the potential well of the nucleus*



[[https://cnx.org/contents/IO\\_vrGwv@3/Nuclear-Binding-Energy](https://cnx.org/contents/IO_vrGwv@3/Nuclear-Binding-Energy)]

*Bethe-Weizsäcker:*

$$E_B = a_v A - a_s A^{2/3} - a_c Z^2 A^{-1/3} - a_A (N - Z)^2 A^{-1} - \delta(N, Z)$$



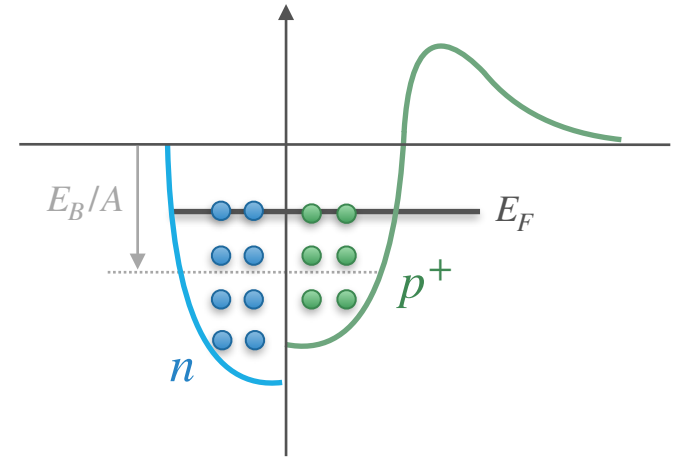
Stable nuclei:

- $N=Z$  for small  $A$
- $N>Z$  for larger  $A$

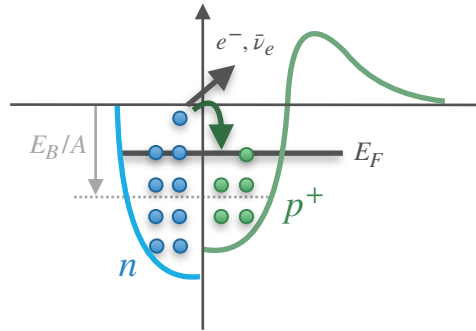
# Nuclear binding models

## Fermi model:

protons and neutrons are Fermi gases in separate potential wells.  
Good description of nucleus in ground state.



## $\beta$ decay in Fermi model



## Shell model:

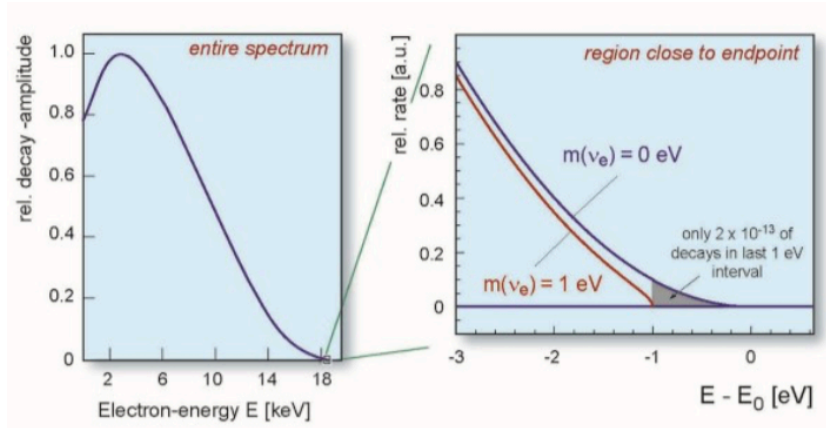
Nucleons take discrete energy eigenstates in Wood-Saxon potential of nucleus.

- Explains magic numbers!
- Compare atomic energy levels:

$n, l$  quantization       $l$ -s coupling

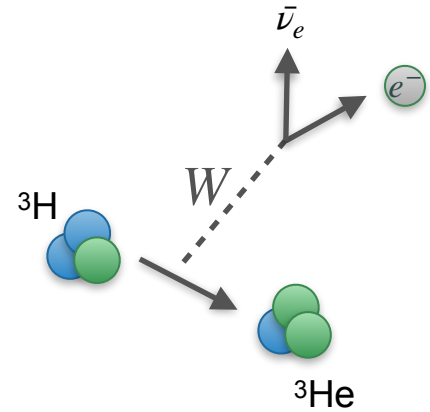
# Tritium decay and the neutrino mass

Curie plot: Beta spectrum of decay electron



Close to endpoint:

Maximal electron energy smaller  
if neutrino has mass



Katrin experiment in Karlsruhe



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# Warm-up question 1

Which statements about the standard model are correct?

A) The Higgs boson can interact with all particles of the SM.

not gluons, photons, neutrino



The CP symmetry is slightly violated in the weak interaction.

Maximal parity violation is implemented in the SM.

C) Parity violation can be observed, but is not part of the Standard Model.



The top quark couples stronger to the Higgs than all other quarks.

The fermion masses arise from the Yukawa term.

E) The fermion masses are a direct consequence of the symmetry breaking in the SM.



## Warm-up question 2

Which statements about atomic nuclei are correct?

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## Warm-up question 2

Which statements about atomic nuclei are correct?



Large, stable nuclei contain more neutrons than protons.

Fusion frees more energy

B) Nuclear fission events free more energy than nuclear fusion events.

C) The nucleons in the deuteron ( ${}^2\text{H}$ ) are stronger bound than the nucleons in the  ${}^{238}\text{U}$  nucleus

The binding energy per nucleon is higher in  ${}^{238}\text{U}$

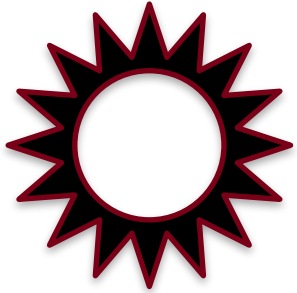
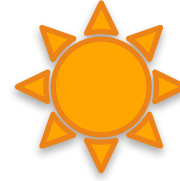
D) The small mass of the neutrino strongly influences the energy spectrum of  $\beta$  radiation.

Only slightly changes shape of tail.

## Add on: Neutrino oscillations

# Neutrinos

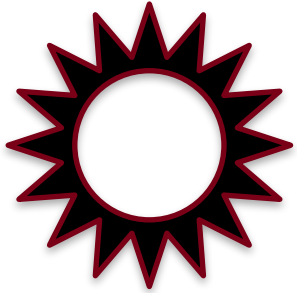
Where do neutrinos come from?



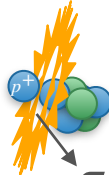
# Neutrinos

Where do neutrinos come from?

Supernova  
explosions



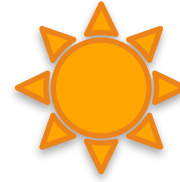
$p^+$



atmospheric  
neutrinos

$\pi$

Big Bang?



Sun (solar neutrinos)

produces only  $\nu_e$

Colliders

nuclear power  
plants



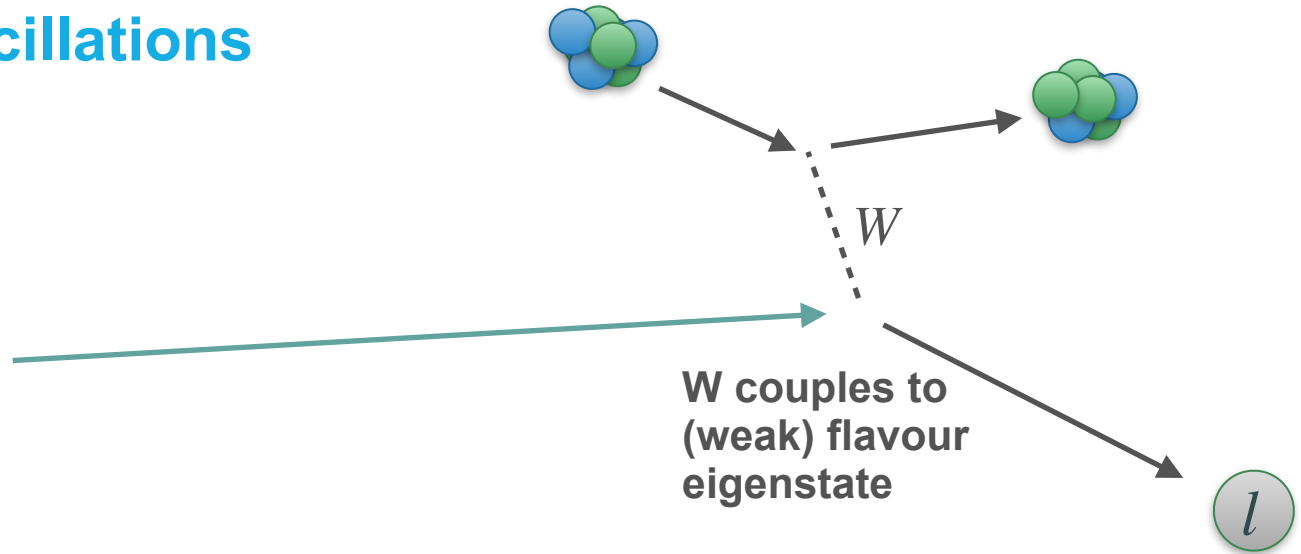
# Neutrino oscillations

constant mass eigenstate



Travelling neutrino is in energy eigenstate (mass eigenstate)

$$m_\nu > 0!$$



# Neutrino oscillations

constant mass eigenstate



Travelling neutrino is in energy eigenstate (mass eigenstate)

$$m_\nu > 0!$$

$$\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

**PMNS matrix**

(compare CKM matrix)



**W couples to (weak) flavour eigenstate**



**Alternative:**

Parameterisation with angle  
(compare Cabibbo / Weinberg angle)