Introduction to Nuclear and Particle Physics

Lesson 1

special relativity

Elemente der Übungsstunde

~~Aufgaben durchrechnen~~

aus Hausaufgaben, Klausuren, Internet

vor allem ihr selbst

Begeisterung + Spass

Videos, fun facts, eure Ideen

Rechentricks + Rezepte

Rechenschritte, Mini-Rechnungen, Probleme

Theorie, Konzepte

Zusammenfassungen, Diskussionen, usw.

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wir gemeinsam

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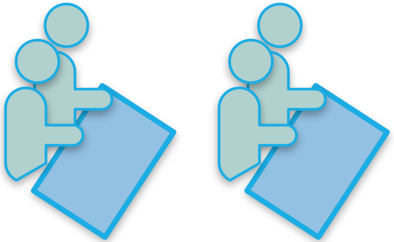
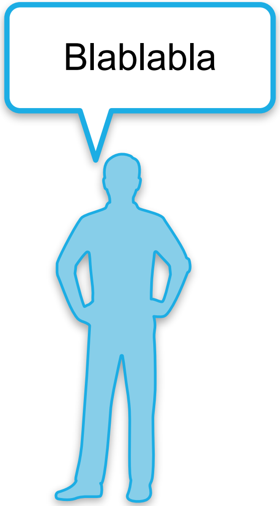
Rolle der Übungsgruppe

Keine Ahnung

Ich will anonym bleiben…





Die Aufgaben kenne ich auswendig

Was kaufe ich nachher ein?

Rolle der Übungsgruppe

Keine Ahnung

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Die Aufgaben kenne ich auswendig

Was kaufe ich nachher ein?

Die Übungsgruppe ist der Ort, an dem ihr nachfragen / euch einbringen / mitgestalten könnt.

Fehler / Unsicherheiten gehören zu jedem Lernprozess und sind nicht peinlich.

Wichtiger Bestandteil:
sozialer Austausch + Interaktion

Overview of the plan for today

mass, energy, momentum

Special Relativity

invariant mass /
center-of-mass energy

Bending radius in
magnetic field

Lorentz transformation, 4-vectors → Time dilation

Warm-up question 1

Which formulas are valid to calculate the kinetic energy of an electron traveling with momentum $p = 1\ \mathrm{GeV}$? (more than 1 possible)

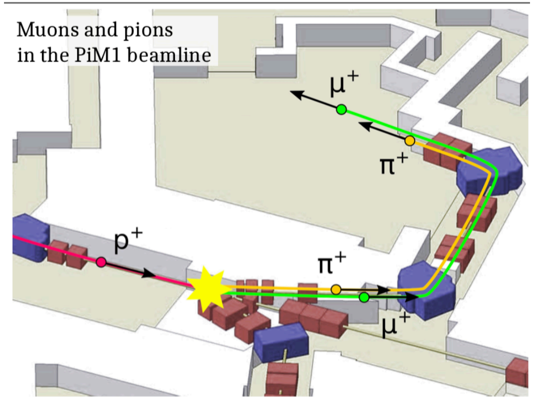
A) $E_{kin} = p$

B) $E_{kin} = \frac{p^2}{2m}$

C) $E_{kin} = \sqrt{m^2 + p^2} - m^2$

D) None of them

Warm-up question 2



Charged particles are moving in a magnetic field.
Which statement is correct?

A) Particles with different mass have the same bending radius as long as their charge and momentum are the same.

B) All particles with the same velocity and charge have the same bending radius.

C) Particles with the same momentum but different mass reach the end at different times.

Remark on exercises with natural units

$c = 2.998 \cdot 10^{8}$ m/s

$e = 1.602 \cdot 10^{-19}$C

$eV = 1.602 \cdot 10^{-19}$J

$\hbar = 1.055 \cdot 10^{-34}$Js

Example:

$$1 \text{ kg} = 1 \text{ N} \cdot \text{s} \cdot \text{s/m}$$

$$1 \text{ kg} = 1 \text{ J/m} \cdot \text{s} \cdot \text{s/m}$$

$$1 \text{ kg} \cdot c^2 = 1 \text{ J} \cdot \left(3 \cdot 10^{8}\right)^2$$

Strategy:

- unit to be expressed should be on the left
- try to “produce” Joule on the right, then use

$$eV = 1.602 \cdot 10^{-19} \text{J}$$

- **set c and h to 1 only in the very end!**

Mass, energy and momentum in special relativity

Relativistic particle

Total energy

$$E = \sqrt{?? + ??}$$

$$E = ??$$

Momentum

$$p = ??$$

rest energy

$$E_0 = mc^2$$

"Equivalence of mass and energy"

kinetic energy

$$E_{kin} = E - ??$$

Non-relativistic particle

$$E_{kin} \ll mc^2$$

Total energy

$$E = mc^2 + \frac{p^2}{2m}$$

Momentum

$$p = mv$$

Relativistic factors

$$\beta = \frac{v}{c} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\beta = 0\ldots1 \qquad \gamma = 1\ldots\infty$$

$$\beta = \frac{pc}{E} \qquad \gamma = \frac{E}{mc^2}$$

Mass, energy and momentum in special relativity

Relativistic particle

Total energy

$$E = \sqrt{m^2c^4 + p^2c^2}$$

$$E = \gamma \cdot mc^2$$

Momentum

$$p = \gamma \cdot mv$$

rest energy

$$E_0 = mc^2$$

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$$E_{kin} = E - E_0$$

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Mass, energy and momentum in special relativity

Relativistic particle

Total energy

$$E = \sqrt{m^2 + p^2}$$

$$E = \gamma \cdot m$$

Momentum

$$p = \gamma \cdot mv$$

rest energy

$$E_0 = m$$

"Equivalence of mass and energy"

kinetic energy

$$E_{kin} = E - E_0$$

Non-relativistic particle

$$E_{kin} \ll m$$

Total energy

$$E = m + \frac{p^2}{2m}$$

Momentum

$$p = mv$$

Relativistic factors

$$\beta = v \qquad \beta = \frac{p}{E}$$

$$\beta = 0 \ldots 1 \qquad \gamma = 1 \ldots \infty$$

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}} \qquad \gamma = \frac{E}{m}$$

Approximations for extreme cases

$$E = \sqrt{m^2 + p^2}$$

Non-relativistic limit: $E_{kin} \ll m$

$$E = m \cdot \sqrt{1 + \frac{p^2}{m^2}} \approx m + \frac{p^2}{2m} + \dots$$

$$E_{kin} = \frac{p^2}{2m} \qquad E_0 = m$$

thermal muons:

$$m_\mu = 105.7\ \mathrm{MeV}$$

$$E_{kin} \approx 40\ \mathrm{meV}$$

Ultra-relativistic limit: $E_{kin} \gg m$

$$E = p \cdot \sqrt{1 + \frac{m^2}{p^2}} \approx p + \frac{m^2}{2p} + \dots$$

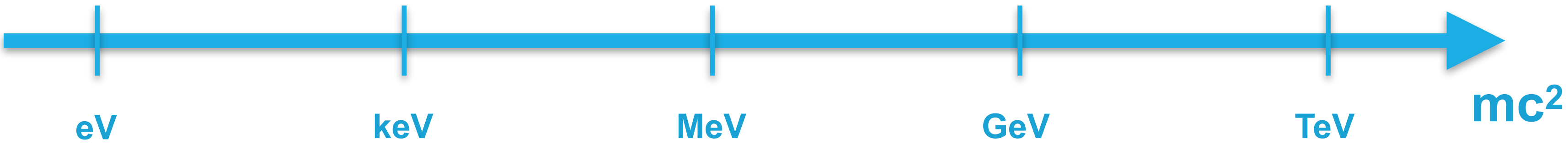
$$p = E = E_{kin}$$

Electrons from μ decay:

$$m_e = 0.511\ \mathrm{MeV}$$

$$E_{kin} \approx 50\ \mathrm{MeV}$$

Some mass scales to know



electron

neutrino

Nucleon

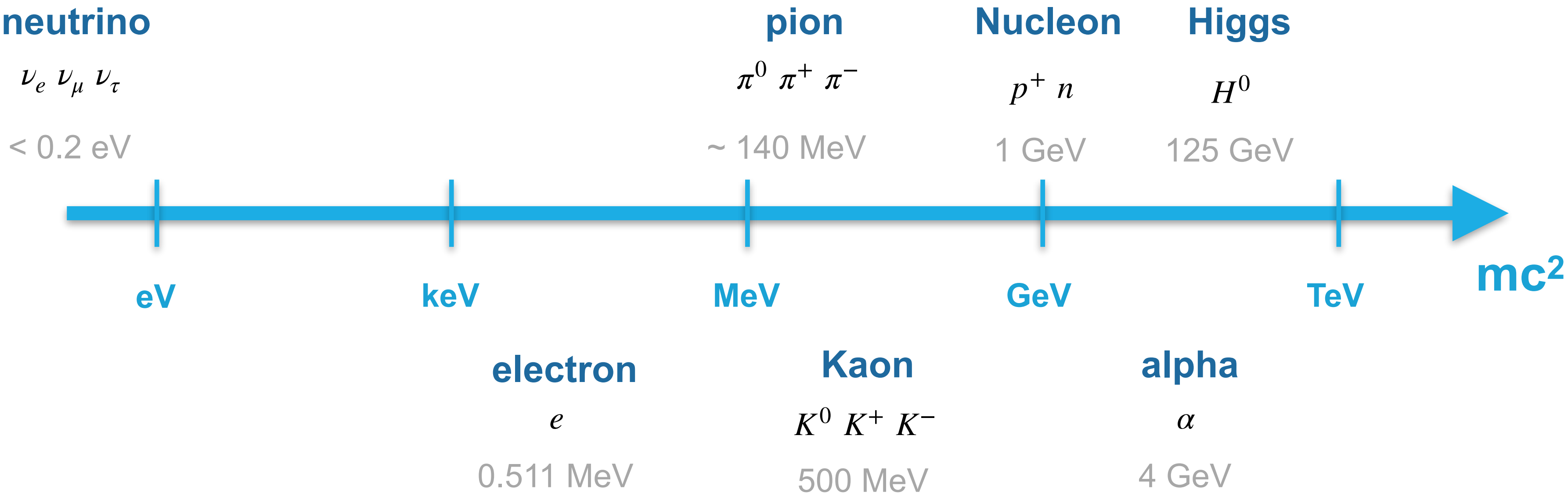
alpha

Higgs

Kaon

pion

Some mass scales to know



Energy of a cosmic muon

A cosmic muon is approaching the surface of the earth with a momentum of $\left|\vec{p}\right| = 1\ \mathrm{GeV/c}.$

What is the energy of the muon in the system of the earth (LAB)?

$$E = ??$$

What is the energy of the muon in the muon system?

$$E' = ??$$

Energy of a cosmic muon

A cosmic muon is approaching the surface of the earth with a momentum of $\left|\vec{p}\right| = 1\ \mathrm{GeV/c}$.

What is the energy of the muon in the system of the earth (LAB)?

$$E = \sqrt{m^2 + p^2} = 1.006\ \mathrm{GeV}$$

What is the energy of the muon in the muon system?

$$E' = \sqrt{m^2 + p'^2} = m = 105.6\ \mathrm{MeV}$$

The total energy changes under Lorentz transformation.
It is **not Lorentz-invariant**.

Recap: Lorentz transformation and four-vectors

In special relativity, Lorentz transformation is needed to change between inertial systems.

How this transformation looks like depends on the transformed object! Two examples:

4 - vectors

Examples:

position vector

$$\mathbf{x} = \begin{pmatrix} ct \\ \vec{x} \end{pmatrix}$$

Physics II

Lorentz scalars

Examples:

Space time interval

$$\mathbf{d} = \sqrt{\mathbf{x}_\mu \cdot \mathbf{x}^\mu} = \sqrt{\mathbf{c}^2\mathbf{t}^2 - \left|\vec{\mathbf{x}}\right|^2}$$

SI units

Recap: Lorentz transformation and four-vectors

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How this transformation looks like depends on the transformed object! Two examples:

4 - vectors

transform with $x' = \Lambda_L \, x$

transformation matrix Λ

Examples:

"boost" in x

$$\Lambda_x = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

position vector

$$\mathbf{x} = \begin{pmatrix} ct \\ \vec{x} \end{pmatrix}$$

Physics II

4-momentum

$$\mathbf{p} = \begin{pmatrix} E/c \\ \vec{p} \end{pmatrix}$$

new

Lorentz scalars

stay unchanged under Lorentz transformation

"lorentz-invariant"

Examples:

Space time interval

$$\mathbf{d} = \sqrt{\mathbf{x}_\mu \cdot \mathbf{x}^\mu} = \sqrt{\mathbf{c}^2\mathbf{t}^2 - \left|\vec{\mathbf{x}}\right|^2}$$

rest mass / invariant mass

$$\mathbf{mc} = \sqrt{\mathbf{E}^2/\mathbf{c}^2 - \left|\vec{\mathbf{p}}\right|^2} = \sqrt{\mathbf{p}_\mu \cdot \mathbf{p}^\mu}$$

natural units

The invariant mass

Note:

- Sum convention: $p_\mu p^\mu = \sum_{\mu=0}^{3} p_\mu p^\mu$
- 4-product introduces minus sign

Definition: $\sqrt{\mathbf{s}} = \sqrt{\mathbf{p}_\mu \cdot \mathbf{p}^\mu} = \sqrt{\mathbf{E}^2 - \mathbf{p}^2}$

$\Rightarrow$ Lorentz-invariant quantity!

single particle:
equal to rest mass

$m = \sqrt{E^2 - p^2}$

several particles:
equal to center-of-mass energy

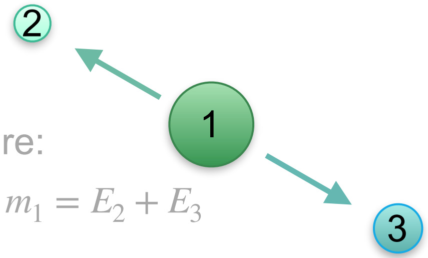
because $\sqrt{s} = E$ for $|\vec{p}| = 0$

Meaning:

- Energy which is available in the center of mass
- Determines total energy of decay or collision products

here:

$m_1 = E_2 + E_3$



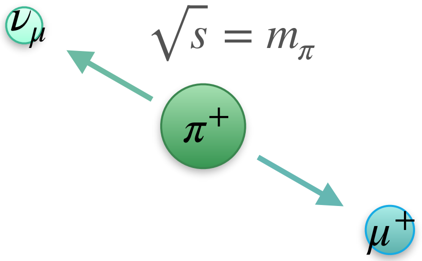
Question about the invariant mass

Positive pions ($m_\pi = 140\ \mathrm{MeV/c}$) decay in most of the cases into a positive muon ($m_\mu = 106\ \mathrm{MeV/c}$) and a neutrino ($m_\nu \approx 0$).

What are the momenta of both muon and neutrino after the decay when a pion decays at rest?

Use 4-momentum conservation: $P_\pi = P_\mu + P_\nu$

$$\begin{pmatrix} E_\pi \\ \vec{0} \end{pmatrix} = \begin{pmatrix} E_\mu + E_\nu \\ \vec{p_\mu} + \vec{p_\nu} \end{pmatrix}$$



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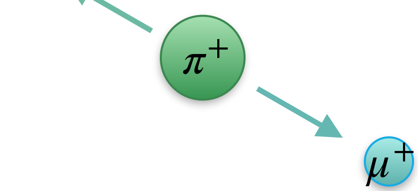
$$\sqrt{s} = m_\pi$$



$$\begin{pmatrix} E_\pi \\ \vec{0} \end{pmatrix} = \begin{pmatrix} E_\mu + E_\nu \\ \vec{p}_\mu + \vec{p}_\nu \end{pmatrix}$$

$$\begin{pmatrix} m_\pi \\ \vec{0} \end{pmatrix} = \begin{pmatrix} \sqrt{m_\mu^2 + \vec{p}_\mu^{\,2}} + |\vec{p}_\nu| \\ \vec{p}_\mu + \vec{p}_\nu \end{pmatrix}$$

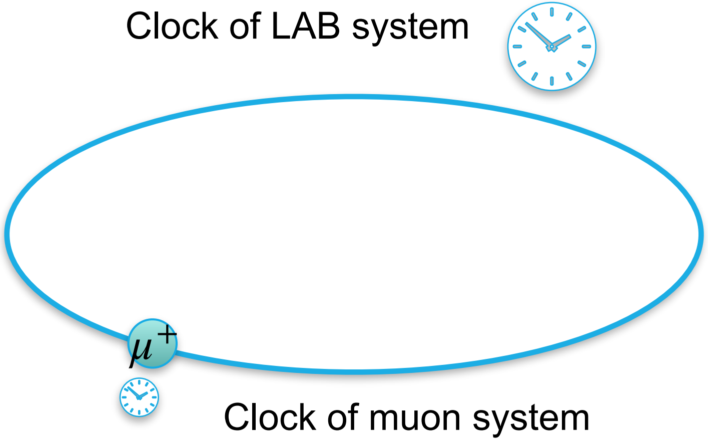
$$m_\pi - p = \sqrt{m_\mu^2 + p^2}$$

$$|\vec{p}_\mu| = |\vec{p}_\nu| = p$$

$$\Rightarrow p = \frac{m_\pi^2 - m_\mu^2}{2m_\pi} = 29.9\ \mathrm{MeV/c}$$

Time dilation

The lifetime of a muon is $\tau = 2.2\ \mu\mathrm{s}$.
Which lifetime τ' would we measure for a muon cycling in a storage ring at $p = 1\ \mathrm{GeV/c}$?



A) $\tau' < \tau$

B) $\tau' \approx \tau$

C) $\tau' > \tau$

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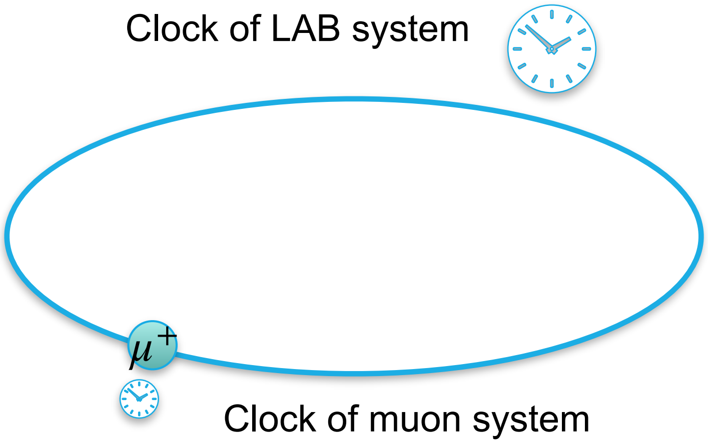


A) $\tau' < \tau$

B) $\tau' \approx \tau$

C) $\tau' > \tau$ ✓

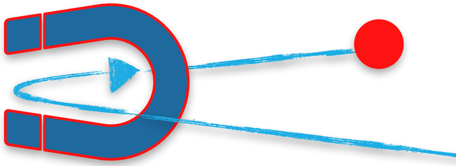
Time dilation! $\tau' = \gamma\tau$
"Time measured in one's rest frame is always shortest."

How do we actually bend particle trajectories?

Picture of the world's most famous accelerator.

(not muons here, but mainly protons)

Hints for calculation of the bending radius



Bending radius of particles moving perpendicular to B field: $$R = \frac{p_\perp}{qB} = \frac{\gamma\ m v_\perp}{qB}$$

Attention: For relativistic particles, Newton II does not hold in the form $F = m \cdot a$

but only in the more general form $F = \frac{\mathrm{d}p}{\mathrm{d}t}$.

Basic idea:

$$F_L = \left| \frac{\mathrm{d}\vec{p}_\perp}{\mathrm{d}t} \right|$$

Lorentz force stays same for relativistic particles

$$F_L = q v_\perp B$$

(no E field)

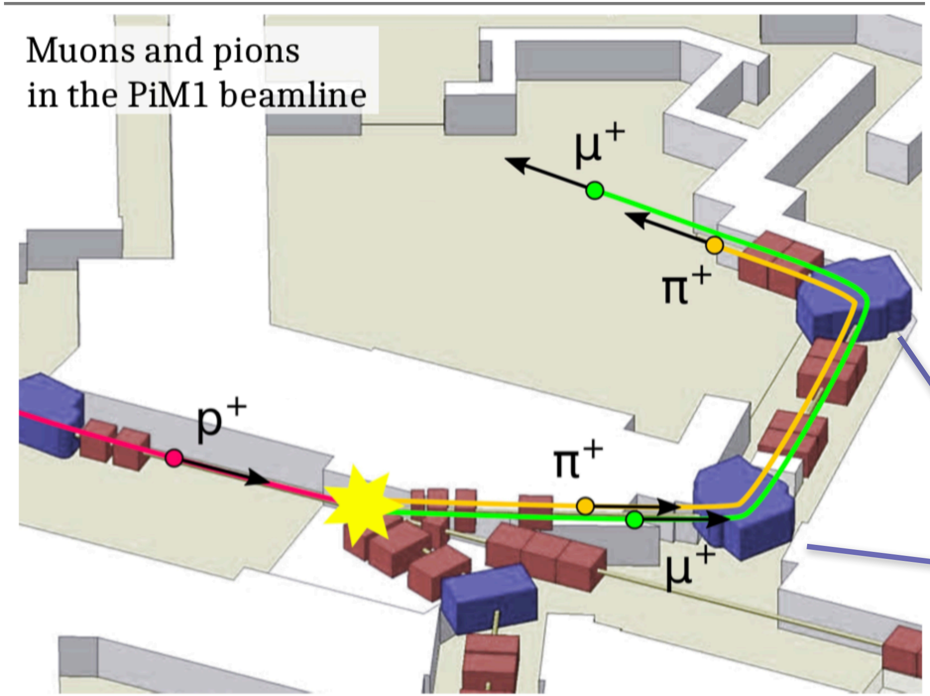
Centripetal term changes!

$$\frac{\mathrm{d}\vec{p}_\perp}{\mathrm{d}t} = \frac{\mathrm{d}\left(\gamma \cdot m\vec{v}_\perp\right)}{\mathrm{d}t}$$

What does circular motion with constant speed tell us about $\frac{\mathrm{d}\gamma}{\mathrm{d}t}$?

Derive $\frac{\mathrm{d}\vec{v}_\perp}{\mathrm{d}t}$ as for non-relativistic particles.

Application: magnets in the PSI beamlines



What is the direction of the B field lines in the blue dipole magnets?

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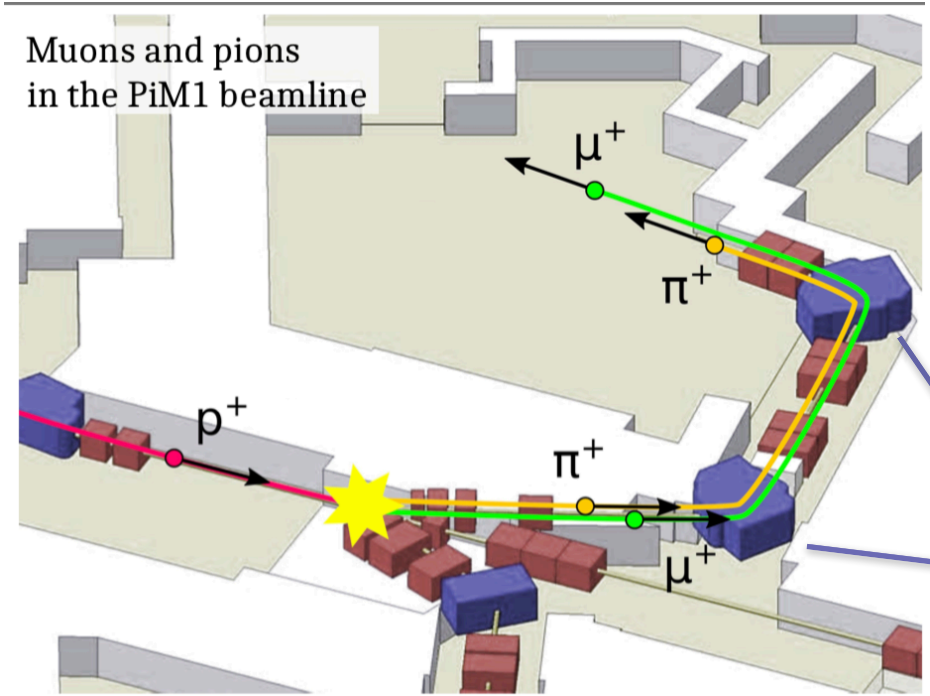

What is the direction of the B field lines in the blue dipole magnets?

right-hand rule

Warm-up question 1

Which formulas are valid to calculate the kinetic energy of an electron traveling with momentum $p = 1\ \mathrm{GeV}$? (more than 1 possible)

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D) None of them

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Which formulas are valid to calculate the kinetic energy of an electron traveling with momentum $p = 1\ \mathrm{GeV}$? (more than 1 possible)

A) $E_{kin} = p$ — Ultra-relativistic limit

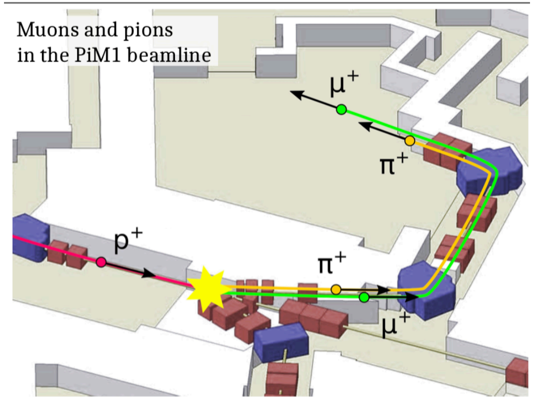
B) $E_{kin} = \dfrac{p^2}{2m}$

These electrons are ultra-relativistic!

C) $E_{kin} = \sqrt{m^2 + p^2} - m^2$ — Precise calculation

D) None of them

Warm-up question 2



Charged particles are moving in a magnetic field.
Which statement is correct?

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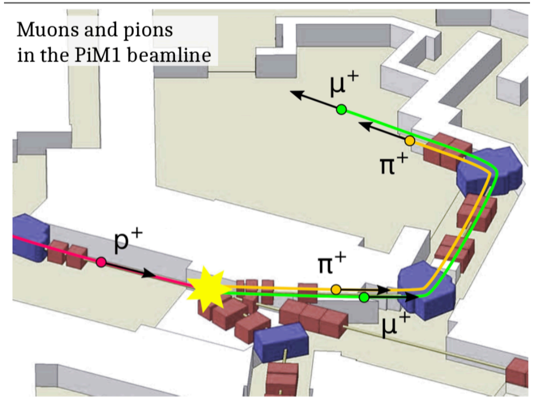

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Which statement is correct?

$$R = \frac{p_\perp}{qB} = \frac{\gamma\ m v_\perp}{qB}$$

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✓ C) Particles with the same momentum but different mass reach the end at different times.

$$v = \frac{p}{\gamma \cdot m}$$